

# Do Political Parties Value Government Portfolios Symmetrically? Evidence from European Parliaments 1965-2018

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## Abstract

While studying government formation in parliamentary democracies, researchers have always assumed that political parties possess identical preferences over government portfolios. This paper shows horizontal differentiation among government portfolios in Western European Parliaments from 1965-2018 in two steps. First, novel empirical patterns show that right-party politicians were more likely to be the Minister of Defense, Minister of Agriculture, Minister of Justice, and Prime Minister. At the same time, the left was more likely to be allocated to Labor, Environment, Health, Science and Technology, Education, and Transport departments. In the second step, party preferences are estimated as the function of their ideology by modeling this strategic interaction as a Colonel-Blotto game. The model provides one with a prediction about "who gets what" which is exploited to uncover party preferences as a function of party ideology. Counterfactual experiments uncover the proportion of allocations that can be explained by heterogeneous preferences and the loss in surplus caused by strategic interactions.

## 1 Introduction

Government formation in parliamentary democracies is one of the most studied topics in political economy and comparative politics ([Baron and Ferejohn, 1989](#); [Diermeier](#)

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et al., 2003; Merlo, 1997; Snyder Jr et al., 2005; Ansolabehere et al., 2005, 2003; Baron, 1993; Laver, 1998). It is conceptualized as a bargaining problem where politicians are interested in obtaining a big proportion of the pie in the least amount of time possible. However, the process is far more complicated than a simple pie division. First, political parties bargain over indivisible goods (government portfolios/departments). Each portfolio has its own functions, duties, and influence over an economy. The Defense Department is responsible for maintaining national security, whereas the Education, Training, and Skills department is dedicated to improving human capital.

Second parties that rarely agree on policies may not agree on which portfolios are more important. The left may value the Education Department over the Defense Department. At the same time, the right may value some other portfolio over Education. In other words, these government portfolios may be horizontally differentiated, which traditional "bargaining over one divisible good" models can not address.

The above-stated hypothesis negates the crucial assumption of "*one pie*", which is present in bargaining models (Baron and Ferejohn, 1989). Such an assumption rules out any Pareto improvement that political parties may benefit from as the models exhibit no delay in equilibrium. Under these conditions, any allocation of this pie will be Pareto optimal if there is no delay. This model restricts us from analyzing suboptimal allocations, and any counterfactual experiment is pointless as no Pareto improvements can be made. However, if there are heterogeneous preferences, we are in the realm of multiple goods. Even under no delay, there can be room for Pareto improvement. Moreover, this can allow us to recommend optimal bargaining protocols to provide better government portfolio allocations.

Even though the government portfolios are individual entities, currently, no theory-backed models can predict "*who gets what?*". This is a severe constraint as it limits us from digging deeper into the functioning of governments. By constructing a framework that can provide us predictions on allocations of these portfolios, we can execute policy counterfactual experiments that may help us improve the government formation process and effectively lower the delay in government formation.

In this article, I provide evidence that parties do not possess identical preferences over government portfolios, and their political ideology is a strong predictor of this heterogeneity. First, I document correlations that show the left has been allocated

certain government portfolios more than the right and vice-versa. For instance, right party politicians were more likely to be a *Minister of Defense*, *Minister of Agriculture*, *Minister of Justice* and *Prime Minister*. Meanwhile, the left party office holders occupied *Labor*, *Environment*, *Health*, *Science and Technology*, *Education*, *Transport* departments more. These correlations are robust after controlling for bargaining weights, country fixed effects, and clustered standard errors. These correlations suggest that left parties may have different preferences over government portfolios than their right counterparts.

I show that these correlations result from heterogeneous preferences and are not a consequence of latent correlations between a party's bargaining power and ideology. In order to answer this, one requires an economic framework that can handle multidimensional bargaining. However, current models do not possess the flexibility to handle such an expansion. I take a different approach to tackle this problem than the literature has done before me.

I consider a model where parties compete across multiple contests or are subject to a Colonel-Blotto game. In this framework, parties obtain utility from a government portfolio determined by their ideology. Each portfolio has a contest where parties report how long they are willing to wait for the portfolio. Once these reports are in, the contest chooses a winner where the contest coefficients are a function of the party's proportion of seats in the coalition. However, in order to bid, the parties must pay a cost. The marginal cost of a bid depends upon the proportion of seats each party has. This dependence allows me to account for the vertical differentiation across government portfolios.

There are two main parallels between the models used by the literature and the framework introduced in this paper. Firstly, the participating players in bargaining protocols indicate their patience by accepting or rejecting an offer. This is accommodated here since parties directly signal how long they are willing to wait for a given portfolio. Second, in bargaining protocols, a party's probability of becoming the proposer is one of the main drivers of surplus extraction. In my model, the marginal cost for parties is lower for parties with higher seats. This resulted in a higher surplus extraction for parties that won more seats in the ruling government.

The model's equilibrium probabilities allow me to form predictions for government

portfolio allocations. I assume a measurement error between the data and the model predictions. I exploit this obtained relation to form a likelihood function and then estimate the model. From the estimation exercise, I find that portfolios corresponding to Agriculture, Defense, Finance, and Justice are sought by the right more than the left. Whereas Labour, Environment, Education, Health, and Transportation are sought more by the left than the right. Either side equally seeks the other portfolios.

I also execute counterfactual experiments that uncover the proportion of cabinet allocations that can be explained by horizontal differentiation. I find that the allocations for portfolios such as Defense, Labour, Agriculture, Environment, Finance, Education, Justice, and Transportation can not be fully explained by just vertical differentiation ([Adachi and Watanabe, 2008](#)) of these portfolios alone. Specifically, one needs horizontal differentiation to explain 22% of allocations for each Defense and Labour portfolio. This amounts to 71 Governments for each portfolio. The lowest proportion of allocations explained by horizontal differentiation is 8.4% for the Finance portfolio. At the same time, horizontal differentiation has no explanatory power for explaining the allocation of PM, Science and Technology, Foreign Relations, General Economic Affairs, and Home Affairs.

I also execute counterfactual experiments that uncover surplus loss arising due to strategic interactions between political parties. For this purpose, I consider two types of social welfare functions. The first considers a naive sum of the pay-off of individual parties. The second considers a weighted sum of the pay-off of individual parties. The weights here are given by the party's proportion of seats in the ruling government.

From the first welfare function, I find a loss of approximately 200%. This is unrealistic since the sum of the pay-offs is higher if there are a higher number of parties, even though they may have a smaller government size. I find a 50% improvement in the parties' welfare from the second welfare function. This improvement is explained by a mix of *better allocations* and *lower delay* in government formation.

This paper contributes to the literature on government formation in parliamentary democracies and introduces a new framework for studying multidimensional and multilateral bargaining. The literature on bargaining and government formation is vast, and I do not pretend to review it exhaustively here. One of the most prominent theoretical paper in this literature [Baron and Ferejohn \(1989\)](#) generalize a Rubinstein game

that accounts for various agreement rules. The first empirical study of government formation was done by [Browne and Franklin \(1973\)](#) and [Browne and Frendreis \(1980\)](#). They assumed that all cabinets have the same weights and are equally valued by everyone. [Warwick and Druckman \(2001\)](#) analyze the relationship between cabinet post allocation and seat shares after using the ranking of the importance of ministers reported by [Laver and Hunt \(1992\)](#).

The first model that allows for a delay in equilibrium is provided by [Merlo and Wilson \(1995\)](#), and it was estimated on the data by [Merlo \(1997\)](#); [Diermeier et al. \(2003\)](#). Authors study western European parliaments and compare delays in government formation across various institutional settings such as a vote of no confidence and investiture.

Authors in [Snyder Jr et al. \(2005\)](#) construct a model where each party's expected pay-off depends on its voting weight. When many high-weight parties existed, then low-weight parties received disproportionately higher pay-off. Their models also support ex-post coalition formateur extracts disproportionately higher pay-off. The authors empirically verify these findings. [Ansolabehere et al. \(2005\)](#) calculate minimal-integer-voting weights for coalition government from 1946-2001 and use these weights to analyze formateur advantages in government formation. [Ansolabehere et al. \(2005\)](#) study the conditions where unequal representation in a bicameral legislature may lead to unequal division of public expenditures.

[Adachi and Watanabe \(2008\)](#) recover the weights of individual ministerial positions by using the uniqueness result from [Eraslan \(2002\)](#) for Baron–Ferejohn games. Where they treat individual portfolios heterogeneously, they do not treat parties' preferences heterogeneously. An examination in support of my argument was done by [Bäck et al. \(2011\)](#), where the authors find that electoral manifestos are strong predictors of determining who wins a government portfolio. My paper micro-founds an empirical approach that has some similarity with the approach used by [Bäck et al. \(2011\)](#). Authors in [Ecker et al. \(2015\)](#) have studied the obtained distribution of government portfolio allocations mechanism suggested by old philosophers.

I contribute to the above literature by providing one of the first empirical examinations that recover distinct party preferences over government portfolios. The model allows for a way to control for vertical differentiation, and also, the model does not

force the Pareto optimal allocation to be identical to the strategic allocation.

The paper proceeds as follows in Section 2 I discuss the data and the summary statistics. In Section 3, I provide empirical patterns obtained using reduced form analysis. I discuss and solve the model in Section 4. In Section 5, I discuss my identification and estimation strategy. In Section 6, I discuss the estimates I obtain from the estimation exercise. In Section 7 I discuss the counterfactual experiments and in Section 8 I conclude the paper.

## 2 Data

### 2.1 Government Portfolio Allocations

For this study I rely on two major data sources. The first data source has been constructed by [Nyrup and Bramwell \(2020\)](#). The dataset contains yearly data on government portfolio allocations for 177 countries during the period 1966–2016. The data source allows one to obtain data on politicians who received any ministerial position in a government. Moreover, it contains the data on the politician's party affiliation. For my study I focus on allocations ministerial position across 16 Government portfolios. These government portfolios include, *Prime Minister Office, Finance, Education, Health, Defense, Agriculture, Foreign Relations, Foreign Economic Relations, General Economic Affairs, Transportation, Agriculture, Labour, Environment, Home Minister's Office, Justice, Planning and Science & Technology*.

### 2.2 Party Ideology Positions

I relied on [Döring and Manow \(2022\)](#) for ideological positions and number of seats won by political parties. There are slight differences in party names. I relied on manual merging wherever there were multiple or no matches between parties (within a country) across the two datasets. As there have been multiple party splits and mergers across Europe, the datasets at times would have disagreement in party names. At times, one of the datasets would refer to the newly (or older) coalition name while the other would use the individual party name. In those situations I relied on a party's history to correctly match the observations.

## 2.3 Summary Statistics

The Table 1 shows the summary statistics for the data. The first three columns shows the summary stats when I consider only cabinets as the unit of observation and. The next three columns shows the summary statistics when one considers coalition parties<sup>1</sup> as the unit of observations. From here on, when I refer to a party, it would mean a coalition party. The scaling of party ideology is between 0 to 10, where 0 denotes the left and 10 denotes the right. On average, a cabinet and a party occupies a center position as the average ideology is close to 5. Moreover, I also show the average size of a party that is a part of the ruling coalition. Generally the ruling party has 70% of the seats in a coalition. I also show the average size of the ruling coalition. The ruling coalition has on average had 69% of the seats. The ruling coalition size here refers to the total proportion of seats the ruling coalition has in a parliament.

I also show the proportion of governments/cabinets that have had a given government portfolio. Apart from the Prime Ministerial cabinet not all cabinets have been present in all governments. At times certain duties of a portfolio are bundled with the Prime Minister or the Home Minister's duties. It is not always feasible to separately find the office holder of these individual duties. In those cases, the cabinets are interpreted as a part of

## 3 Evidence for Horizontal Differentiation

In this section I show evidence for the existence of horizontal differentiation of government portfolios across party ideology. First consider the Figure 1 that plots a linear fit of allocation probability over party ideology. From these fits I find that portfolios such as (i) Defense, Military & National Security, (ii) Agriculture, Food, Fisheries & Livestock and (iii) Justice & Legal Affairs show significant and positive raw correlation between party ideology and the allocation probability. These patterns indicates these portfolios are more likely to be allocated to the right than the left.

An examination of the linear fits for portfolios such as (i) Labor, Employment & Social Security (ii) Environment and (iii) Labor, Employment & Social Security and (iv)

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<sup>1</sup>Coalition party here means those parties that are a part of the ruling coalition.



Table 1: Summary Stats

	Government/ Cabinets			Cabinet Parties		
	Mean	Std.	N	Mean	Std.	N
Ideology	5.4343971	0.8914147	356	5.3774046	1.7186277	1245
Party's Proportion of Seats (within Cabinet)				0.7158778	0.1674270	1278
Cabinet's Proportion of Seats	0.6920951	0.1736291	356			
Prime Minister	1.0000000	0.0000000	356	0.2785603	0.3933847	1278
Agriculture, Food Fisheries and Livestock	0.9157303	0.2781828	356	0.2550861	0.3832544	1278
Defense, Military and National Security	0.8904494	0.3127683	356	0.2480438	0.3747589	1278
Education, Training and Skills	0.9101124	0.2864233	356	0.2535211	0.3665497	1278
Environment	0.7921348	0.4028699	356	0.2206573	0.3403952	1278
Finance, Budget and Treasury	0.9578652	0.2011795	356	0.2668232	0.3661378	1278
Foreign Economic Relations	0.6804775	0.4663820	356	0.1895540	0.3348371	1278
Foreign Relations	0.9618446	0.1850415	356	0.2679317	0.3641790	1278
General Economic Affairs	0.6137640	0.4868483	356	0.1709703	0.3292844	1278
Home Affairs	0.8301899	0.3734523	356	0.2312579	0.3583470	1278
Health and Social Welfare	0.8651685	0.3420242	356	0.2410016	0.3588031	1278
Justice and Legal Affairs	0.9283708	0.2568688	356	0.2586072	0.3906913	1278
Labour, Employment and Social Security	0.8539326	0.3536709	356	0.2378717	0.3636081	1278
Planning and Development	0.5505618	0.4981371	356	0.1533646	0.3161611	1278
Science, Technology and Research	0.6376404	0.4813585	356	0.1776213	0.3329971	1278
Transport	0.7724719	0.4198263	356	0.2151800	0.3593963	1278

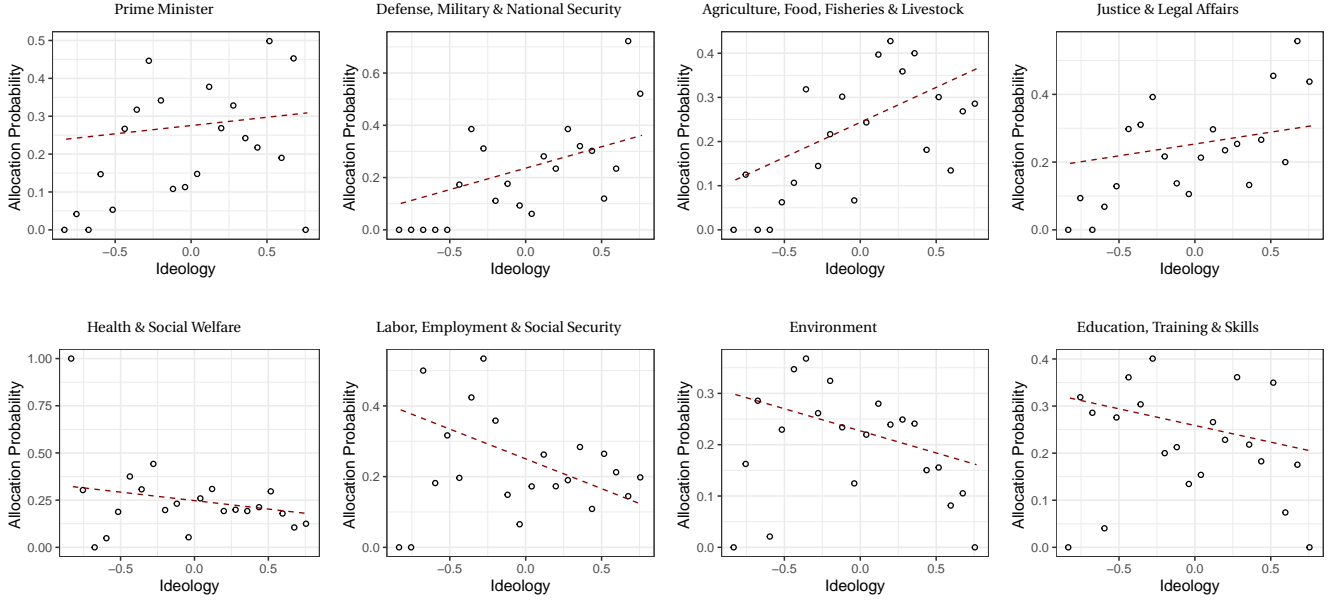
Education, Training & Skills— indicates that these portfolios are more likely to be allocated to the left than to the right.

While the Figure 1 shows raw correlations we want to see if these correlations will survive once we control for party size within a coalition country fixed effects and so on. In order to examine this I run the following regressions separately for each government portfolio  $k$ .

$$G_{ic}^k = \beta_0^k + \beta_1^k I_i + \beta_2^k P_{ic} + \gamma_{C(c)} + \epsilon_{ikc} \quad (3.1)$$

In the above regression  $G_{ikc}$  is an indicator variable which takes the value of 1 if a party  $i$  was allocated government portfolio  $k$  in cabinet (or government)  $c$ . The coefficient  $\beta_0^k$  is  $k$  specific fixed effect. The variable  $I_i$  is party  $i$ 's ideology. The coefficient  $\beta_1^k$  is a government portfolio  $k$ -specific slope term associated with party ideology. The term  $P_{ic}$  is the proportion of seats, party  $i$  has in cabinet  $c$  within the proto-coalition. From here on I will refer to this term simply as proportion of seats, however in here the proportion of seats will refer to the proportion of seats within the ruling proto-coalition. The associated  $k$ -specific slope coefficient is indicated by  $\beta_2^k$ . The country fixed effect





**Figure 1:** This Figure plots a bin scatter plot of portfolio allocation outcome,  $G_{ikc}$ , against party Ideology,  $I_i$ . Portfolios such as (i) Defense, Military & National Security, (ii) Agriculture, Food, Fisheries & Livestock and (iii) Justice & Legal Affairs show significant and positive raw correlation between party ideology and the allocation probability. That indicates these portfolios are more likely to be allocated to the right than the left. While portfolios such as (i) Labor, Employment & Social Security (ii) Environment and (iii) Labor, Employment & Social Security and (iv) Education, Training & Skills indicating these portfolios are more likely to be allocated to the left than to the right.

is denoted by  $\gamma_{C(c)}$ . Lastly  $\epsilon_{ikc}$  is the error term for this regression.

Running a separate regression allows me to control for country specific fixed effects and also cluster the standard errors at the country level. As the error term might be heteroskedastic for each portfolio within each country. In this regression I am penalizing the coefficients by keeping degrees of freedom low.

The results from these regressions are shown in Figure 2. Note that the estimated ideology coefficients,  $\beta_1^k$ , for portfolios such as Defense, Agriculture, Justice Prime Minister and Finance are positive significant at 10% level of significance. These correlations survive after controlling for party size and country fixed effects. This indicates that conditioning on party size these portfolios are more likely to go to a right leaning party than a left leaning one.

On the other side of the aisle, the estimates of ideology coefficients for portfolios such as Science and Technology, Transport, Education, Health, Environment and Labour are negative and significant at 90% level of significance. Since we control for party size



**Figure 2:** This figure shows ideology slope estimate for the regressions described in equations 3.1 and 3.2. Coefficient estimates corresponding to specific coefficients point to regressions 3.1 which are portfolio specific. There are a total 1278 observations in each regression where each observation is a party that was part of a coalition  $c$ . Country fixed effects are present here and the standard errors are clustered at the country level. Coefficients corresponding to pooled point to regression 3.2 where cabinet $\times$ portfolio fixed effects are present and the standard errors are also clustered at this level. There are 20,448 observations here.

within the ruling coalition, these indicate that conditional on party size these portfolios are more likely to be allocated to a left leaning party within the ruling coalition.

I also analyze if these patterns hold within cabinet $\times$ portfolio comparisons. For this I run the following pooled regression.

$$G_{ikc} = \gamma_0 + \sum_{l=1}^K \beta_l I_i \times \mathbb{1}\{l = k\} + \sum_{l=1}^K \alpha_l P_{ic} \times \mathbb{1}\{l = k\} + \gamma_{ck} + \epsilon_{ikc} \quad (3.2)$$

Here  $G_{ikc}$  indicates whether portfolio,  $k$ , was allocated to party,  $i$ , in cabinet,  $c$ , or not.  $I_i$  is party  $i$ 's ideology and  $P_{ic}$  is proportion of parliamentary seats within ruling coalition. Moreover,  $\gamma_{ck}$  indicates the cabinet $\times$ portfolio fixed effect. The unobserved term is denoted by  $\epsilon_{ikc}$ .

The results for regression described in equation 3.2, along with its variations, is provided in table 2. Here column (1) is the pooled regression where I allow for only port-

Table 2: Pooled Regression Results

Dependent Variable: Model:	(1)	(2) $G_{ikc}$	(3)
<i>Variables</i>			
Agr × ideology	0.1553*** ( $1.51 \times 10^{-15}$ )	0.1549*** (0.0240)	0.1420*** (0.0240)
Def × ideology	0.1618*** ( $6.55 \times 10^{-16}$ )	0.1614*** (0.0265)	0.1628*** (0.0268)
Edu × ideology	-0.0735*** ( $6.42 \times 10^{-17}$ )	-0.0739*** (0.0263)	-0.0749*** (0.0256)
Env × ideology	-0.0882*** ( $1.04 \times 10^{-16}$ )	-0.0886*** (0.0254)	-0.0821*** (0.0247)
Fin × ideology	0.0420*** ( $3.43 \times 10^{-16}$ )	0.0416 (0.0274)	0.0379 (0.0272)
ForEco × ideology	-0.0413*** ( $3.33 \times 10^{-16}$ )	-0.0417 (0.0257)	-0.0311 (0.0257)
ForRel × ideology	-0.0172*** ( $4.25 \times 10^{-17}$ )	-0.0176 (0.0282)	-0.0083 (0.0278)
GEA × ideology	0.0446*** ( $1.35 \times 10^{-16}$ )	0.0442* (0.0241)	0.0534** (0.0238)
Health × ideology	-0.0922*** ( $2.22 \times 10^{-16}$ )	-0.0926*** (0.0291)	-0.0908*** (0.0288)
Home × ideology	-0.0386*** ( $1.19 \times 10^{-16}$ )	-0.0390 (0.0274)	-0.0389 (0.0273)
Just × ideology	0.0675*** ( $5.6 \times 10^{-16}$ )	0.0671** (0.0290)	0.0727** (0.0287)
Labour × ideology	-0.1712*** ( $6.83 \times 10^{-16}$ )	-0.1716*** (0.0273)	-0.1736*** (0.0264)
Plan × ideology	-0.0228*** ( $5.46 \times 10^{-17}$ )	-0.0232 (0.0228)	-0.0234 (0.0224)
PM × ideology	0.0394*** ( $3.28 \times 10^{-16}$ )	0.0390 (0.0241)	0.0130 (0.0243)
SciTech × ideology	-0.0437*** ( $5.85 \times 10^{-17}$ )	-0.0442* (0.0254)	-0.0466* (0.0249)
Trans × ideology	-0.0567*** ( $2.14 \times 10^{-16}$ )	-0.0571* (0.0300)	-0.0535* (0.0297)
<i>Fixed-effects:</i>	Portfolio	Cabinet + Portfolio	Cabinet × Portfolio
<i>Clustered S.E. :</i>	Portfolio	Cabinet	Cabinet × Portfolio
<i>Fit statistics</i>			
R <sup>2</sup>	0.23279	0.24910	0.24490
Observations	20,448	20,448	20,448

Signif. Codes: \*\*\*: 0.01, \*\*: 0.05, \*: 0.1

<sup>a</sup> The results for regression described in equation 3.2, along with its variations, is provided in table 2. Here column (1) is the pooled regression where I allow for only portfolio fixed effect and cluster standard error at portfolio level. Then columns (2) allows for two-way fixed effects corresponding cabinets and portfolio. Standard errors are clustered at cabinet level here. Column (3) uses cabinet×portfolio fixed effects and the standard errors are at cabinet×portfolio level.

folio fixed effect and cluster standard error at portfolio level. Then columns (2) allows for two-way fixed effects corresponding cabinets and portfolio. Standard errors are clustered at cabinet level here. Column (3) uses cabinet×portfolio fixed effects and the standard errors are at cabinet×portfolio level.

It is important to notice that once we start making comparisons within cabinet and portfolio, that is allow for cabinet × portfolio fixed effects, the correlations between ideology and allocation probability changes for certain portfolios. For instance under these comparisons PM’s coefficient on ideology is indistinguishable from 0 where it was significant for the PM specific regression. The coefficient for GEA is significant now.

Note that allocations are endogenous variables and running an analysis similar to what we have here is not sufficient to purely isolate the preferences of parties. More-

over there are dependence across portfolios that can not be captured in these regressions. Moreover, the number of independent observations here should be cabinets rather than the number of party $\times$ portfolio pairs that we construct.

## 4 Model

From the discussion in the previous section, note that we wish to build a model that treats each government/cabinet as a single observation. Therefore, unlike the regression analysis, the model will not overcount the degrees of freedom. The model needs to address strategic interaction across parties in the ruling coalition. Therefore it needs to be a game and not just a decision theoretic. Moreover, it should make within portfolio comparisons to isolate party preferences.

Keeping the above requirements in mind, I construct a Colonel Blotto game. Each territory of the Colonel Blotto game represents a government portfolio. The effort represents the willingness to wait for a party and they must pay a cost such that the marginal cost is decreasing in proportion of seats the party has. This captures three important features. First, modeling this game as a Colonel Blotto game is a direct exploitation of the parallels that game like war of attrition has with auction based games.

In games similar to war of attrition or even bargaining games the real cost is determined by the delay in coming to an agreement. Here we model that feature directly into strategy space by allowing players to bid waiting times. Moreover, in these games, especially war of attrition, every participating entity pays the cost of delay irrespective of they win or not. This feature is directly mapped in the following game. Lastly, it has been documented that parties with higher share of seats in ruling governments extract higher surplus (Fujiwara and Sanz, 2020; Diermeier et al., 2003; Merlo, 1997; Adachi and Watanabe, 2008). By modeling the marginal costs as decreasing functions of share of cabinet seats I can support this feature within equilibrium behavior.

### 4.1 Preliminary

The objective of the model is to allow for heterogeneous preferences over government portfolios. Traditional bargaining models, which are a natural candidate here, fail to accommodate preference heterogeneity and also multiple indivisible goods. There-

fore, we move away from that and consider a Colonel Blotto game to model this strategic interaction.

There are  $N_c$  parties in a coalition. Members of the coalition partakes in multiple contests simultaneously. These contests are denoted by  $k = 1, 2, \dots, K$  and each contest decides the winner of a portfolio  $k$ . Each portfolio can potentially have heterogeneous benefit across the members and across each other.

In the Colonel Blotto game, proto-coalition members are subject to a budget constraint where each is endowed with  $\delta$  units of maximal budget. Each party  $i$  can choose  $T_{ik}$  units to bid on portfolio  $k$ . However, they must pay a price to do so. This price depends on the the portfolio  $k$  and proportion of seats,  $P_i$  party  $i$  has.

In the event party  $i$  wins the portfolio it receives  $u_{ik}$  unit of utility. This utility depends on party ideology,  $I_i$  and portfolio  $k$ . Below I provide more details on the model.

## 4.2 Party Maximization Problem

In this section I define party  $i$ 's maximization problem. For this first I define the pay-off party  $i$  receives when it wins portfolio  $k$ . Parties obtain  $u_{ik}$  units of utility when they win a portfolio in the corresponding contest. This pay-off is defined as below:

$$u_i(I_i; k) = \exp \{ \omega_0 + \alpha_k I_i \} \quad (4.1)$$

Each portfolio has a base pay-off of  $\omega_0$ . Depending upon party ideology,  $I_i$ , and the sign of  $\alpha_k$ , right-inclined members may have higher/lower utility than the base pay-off. The same is true for left-inclined members. As we know, coalitions which have smaller proportion of seats are more likely to serve shorter terms than the ones that have higher proportion of seats. Benefits from portfolios assigned in fragile coalitions may be lower. To address this issue I allow that portfolio-specific benefits to vary with coalition size.

Each member must pay a cost for participating in the contest irrespective of whether they win or not. The cost is defined as:

$$C_i(T_{ik}; k, P_i) = \exp \{ -\gamma_k P_i \} T_{ik} \quad (4.2)$$

Here the cost that party  $i$  must pay to bid its willingness to wait for portfolio  $k$  has a coefficient that depends upon its seat share in the coalition. Parties that have a higher proportion of seats will have lower prices to pay and therefore will be able to compete more aggressively for respective portfolios. This addresses the feature bargaining models will have, i.e., parties with higher proportion of weights have higher bargaining power and are able to extract more surplus for themselves. I allow for lower costs for such parties. These lower costs will enable the party with rent-extraction which is the isomorphic implication of higher bargaining power.

We assume a Tullock contest function of the following nature:

$$p_{ik}(T_{ik}, T_{-i,k}) = \frac{T_{ik}}{\sum_{j=1}^{N_c} T_{jk}} \quad (4.3)$$

The above contest function ensures that the expected pay-of functions will be strictly concave and we will have unique best responses. The number of choice variables for each player is  $K$ , adding further non-linearities will make the computationally infeasible. Therefore we restrict to the case where exponents and coefficients on these bids are 1.

Each party  $i$  solves the following problem. They maximize their expected payoff with respect to the above specific budget constraint.

$$\begin{aligned} \max_{T_{i1}, T_{i2}, \dots, T_{iK}} \quad & \sum_{k=1}^K \left( \exp\{\omega_0 + \alpha_1 I_i\} \frac{T_{i,k}}{\sum_{j=1}^{N_c} T_{j,k}} - \exp\{-\gamma_k P_i\} T_{i,k} \right) \\ \text{s.t.} \quad & T_{i,k} \geq 0 \quad \forall k = 1, 2, \dots, K \end{aligned} \quad (4.4)$$

### 4.3 Equilibrium Characterization

In this game we search for the Nash equilibrium. Here the Nash Equilibrium is defined as followed:

**Definition 4.1** *A Nash equilibrium in this game is defined by the nested Tuple:*

$$\left\{ \left\{ T_{1k}^* \right\}_{k=1}^K, \left\{ T_{2k}^* \right\}_{k=1}^K, \dots, \left\{ T_{Nk}^* \right\}_{k=1}^K \right\}$$

where  $N$  is the number of parties in the proto-coalition and the following holds:

$$\begin{aligned} \{T_{1k}^*\}_{k=1}^K &\in \arg \max \left\{ \sum_{k=1}^K \left( \exp\{\omega_0 + \alpha_1 I_1\} \frac{T_{1,k}}{\sum_{j=1}^{N_c} T_{j,k}} - \exp\{-\gamma_k P_1\} T_{ik} \right) \text{ s.t. } T_{1k}^* \geq 0 \right\} \\ \{T_{2k}^*\}_{k=1}^K &\in \arg \max \left\{ \sum_{k=1}^K \left( \exp\{\omega_0 + \alpha_1 I_1\} \frac{T_{1,k}}{\sum_{j=1}^{N_c} T_{j,k}} - \exp\{-\gamma_k P_i\} T_{ik} \right) \text{ s.t. } T_{ik}^* \geq 0 \right\} \\ &\vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \\ \{T_{Nk}^*\}_{k=1}^K &\in \arg \max \left\{ \sum_{k=1}^K \left( \exp\{\omega_0 + \alpha_1 I_1\} \frac{T_{1,k}}{\sum_{j=1}^{N_c} T_{j,k}} - \exp\{-\gamma_k P_{N_c}\} T_{ik} \right) \text{ s.t. } T_{N_c k}^* \geq 0 \right\} \end{aligned}$$

Clearly to solve for the Nash equilibrium we will have to solve for  $N \times K$  variables. However, note that the objective functions are strictly concave for all players. This gives us unique best responses. Now I characterize the *Nash Equilibrium*. First consider the first order condition for party  $i$  with respect  $T_{ik}$ , it is given by the following:

$$\exp\{\omega_0 + \alpha_k I_i\} \cdot \left( \frac{T_k - T_{ik}}{T_k^2} \right) = \exp\{-\gamma_k P_i\} \quad (4.5)$$

Here  $T_k$  is defined as  $T_k = \sum_{i=1}^K T_{ik}$ . The above equation holds for all  $i = 1, \dots, N$ . With some manipulation we can obtain tractable expressions for  $T_k$  as functions of lagrange multipliers. We can characterize  $T_k$  as followed:

$$T_k = \frac{N - 1}{\sum_{i=1}^N \exp\{-\omega_0 - \alpha_k I_i - \gamma_k P_i\}} \quad (4.6)$$

This expression provides us with a way of obtaining  $T_{ik}$  as a function of  $T_k$ . Then  $T_{ik}$  is given by the following equation:

$$T_{ik} = \max\{0, T_k \cdot (1 - T_k^* \exp\{-\omega_0 - \alpha_k I_i - \gamma_k P_i\})\} \quad (4.7)$$

Note that  $T_{ik} > 0$  as long as  $1 > T_k \lambda_i \exp\{-\omega_0 - \alpha_k I_i - \gamma_k P_i\}$ . Now in order to solve for equilibrium, we do not need to solve for simultaneous equations. This gives reduced form formulae for  $T_{ik}$  in terms of parameters.

Therefore we are ready to characterize the equilibrium of the model as followed. We utilize the expressions for  $T_k$  and  $T_{ik}$  and substitute it into the budget constraints.

**Proposition 4.1 (Characterization of Equilibrium)** *The Nash Equilibrium is characterized as followed. The aggregate equilibrium bid,  $T_k^*$  is given by the following:*

$$T_k^* = \frac{N - 1}{\sum_{i=1}^N \exp\{-\omega_0 - \alpha_k I_i - \gamma_k P_i\}} \quad (4.8)$$



The  $i$ -specific bid is given by the following equation.

$$T_{ik}^* = \max\{0, T_k \cdot (1 - T_k^* \exp\{-\omega_0 - \alpha_k I_i - \gamma_k P_i\})\} \quad (4.9)$$

Provided these equilibrium bids, we can find the equilibrium allocation probabilities. In our empirical application, it is the allocation probability that provides us with a prediction for the sample that we discussed.

$$p_{ik}(T_{i,k,c}^*, T_{-i,k,c}^*) = \max\left\{0, \frac{T_{ikc}^*}{\sum_{j=1}^{N_c} T_{jkc}^*}\right\} = \max\left\{0, \left(1 - \frac{(N_c - 1) \exp\{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp\{-\alpha_k I_j - \gamma_k P_j\}}\right)\right\} \quad (4.10)$$

Note there are other methods which authors have employed in analyzing the distribution of government portfolio allocations. Few of these use a sequential game approach [Ecker et al. \(2015\)](#); [O'Leary et al. \(2005\)](#) where some traditional methods such as d'Hondt or Saint-Lague, are studied. I bypass the need of specifying a dynamic game by modeling the strategy space directly as the number of periods one would have to wait. Here I am exploiting the type of symmetries war of attrition and all pay auctions posses.

## 5 Identification and Estimation

The parameter  $\omega_0$  is not identified. This parameter changes bids of all players within a cabinet proportionately. Since I use allocation data alone and therefore the parameters that do not change relative bids of parties within a cabinet  $\times$  portfolio are not identified here. As it turns out any portfolio specific fixed effect is not identified here either. Therefore a direct method that allows for vertical differentiation across portfolios is not identified in this game.

Since the parameters that affect horizontal differentiation change relative bids of parties, they also change allocation probabilities. This results in ensuring that horizontal differentiation parameters are identified. In [Adachi and Watanabe \(2008\)](#) parties that posses a higher proportion of seats have a positive correlation with being assigned a more valuable portfolios. Here we can use the proportion of seats directly back out the marginal costs that parties face while bidding for portfolios. These parameters pro-

vide us with a way to control for vertical differentiation. This arguably provides with unbiased estimate for parameters of horizontal differentiation.

This correlation between a party's proportion of seats and the surplus extraction is also documented in [Diermeier et al. \(2003\)](#); [Merlo \(1997\)](#). Here, I exploit this known relationship to control for the existence of vertical differentiation across government portfolios.

Our unit of observation is a coalition government,  $c$ . We observe the allocation of a government portfolio,  $k$ , to a party,  $i$ , in government cabinet  $c$ . We also observe party ideology  $I_{ic}$  and also the proportion of seats party  $i$  has in coalition government  $c$ . The set of observations is given by:  $\left\{ \left\{ \{G_{ikc}\}_{k=1}^K, I_i, P_{ic} \right\}_{i=1}^N \right\}_{c=1}^C$ .

I assume that the data is explained by the model with some added measurement error. Let the error associated with allocation of  $k$  to  $i$  in  $c$  be denoted by  $\epsilon_{ikc}$ . I assume this noise is normally distributed with mean 0 and standard deviation  $\sigma$ . Below I state the non-linear regression that I estimate:

$$G_{ikc} = p_{ik}(T_{i,k,c}^*, T_{-i,k,c}^*) = \max \left\{ 0, \frac{T_{ikc}^*}{\sum_{j=1}^{N_c} T_{jkc}^*} \right\} + \epsilon_{ikc} \quad (5.1)$$

$$\Rightarrow G_{ikc} = \max \left\{ 0, \left( 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right) \right\} + \epsilon_{ikc}$$

Let  $T_{ikc}^*$  be given by proposition 4.1. Then the likelihood of  $k$  allocated to  $i$  is given by:

$$\ell_{kc} = \frac{1}{\sigma^K} \prod_{i=1}^{N_c} \phi \left( \frac{1}{\sigma} \left\{ G_{ikc} - p_i(T_{i,k,c}^*, T_{-i,k,c}^*) \right\} \right) \quad (5.2)$$

$$\Rightarrow \ell_{kc} = \frac{1}{\sigma^K} \prod_{i=1}^{N_c} \phi \left( \frac{1}{\sigma} \left\{ G_{ikc} - \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \right\} \right)$$

Here, we are assuming that the measurement error is uncorrelated across government portfolios. I execute robustness tests where we allow for cabinet specific measurement error shocks. These can capture correlations. The likelihood of the whole cabinet is

given by the following likelihood

$$\ell_c = \frac{1}{\sigma^{N_c K}} \prod_{k=1}^K \prod_{i=1}^{N_c} \phi \left( \frac{1}{\sigma} \left[ G_{ikc} - \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \right] \right) \quad (5.3)$$

This gives the log-likelihood for the whole sample:

$$\ell \ell \left( \theta; \left\{ \left\{ G_{ikc} \right\}_{k=1}^K, \left\{ I_i, P_{ic} \right\}_{i=1}^N \right\}_{c=1}^C \right) = \sum_{k=1}^K \sum_{i=1}^{N_c} \log \left[ \phi \left( \frac{1}{\sigma} \left[ G_{ikc} - \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \right] \right) \right] - \sum_{c=1}^C N_c K \log \sigma \quad (5.4)$$

The number of observations here is given  $C = 356$ . For standard errors I calculate observation wise jacobians and the hessian. In the following section I discuss the obtained results.

## 6 Results

In this section I will discuss the estimates for model parameters and also the estimates for average marginal effects of party ideology and proportion of cabinet seats. First consider Table 3, here I show the point estimates and the standard errors for the parameter estimates. First consider the coefficients of proportion of cabinet seats. Coefficient corresponding to the PM seat is the highest. This indicates that the PM seats are one of the most value seats as it displays the strongest correlation with proportion of cabinet seats.

The second seat to follow PM is the Agriculture Minister's seats. The coefficient is 1.97 and therefore it exhibits the second highest correlation after PM's seat. It is followed by the Finance Minister's seat at 1.58 and then by the Labour Minister's seat. Note that a lot of these coefficients are not statistically indistinguishable from each other. Especially from the Finance Minister's seat.

Note the coefficient on proportion of seats for the PM is disproportionately higher. This speaks to the result obtained in [Fujiwara and Sanz \(2020\)](#) where authors show that the party that obtains the highest proportion of seats has a discontinuously higher chance of acquiring the PM's seat. My estimates support this result by providing a disproportionately higher estimate for PM coefficient associated with proportion of cabinet seats.

Table 3: Parameter Estimates

		Vertical Differentiation Coefficients															
Parameter	$\sigma$	$\gamma_{PM}$	$\gamma_{Agr}$	$\gamma_{Def}$	$\gamma_{Edu}$	$\gamma_{Env}$	$\gamma_{Fin}$	$\gamma_{ForEco}$	$\gamma_{ForRel}$	$\gamma_{GEA}$	$\gamma_{Home}$	$\gamma_{Health}$	$\gamma_{Just}$	$\gamma_{Labour}$	$\gamma_{Plan}$	$\gamma_{SciTech}$	$\gamma_{Trans}$
Est	0.322	4.83***	1.97***	1.2***	1.24***	0.818***	1.58***	0.693***	1.06***	0.36***	1.2***	1.17***	1.17***	1.42***	0.662***	0.812***	0.85***
	0.00377	0.408	0.152	0.112	0.1	0.0855	0.107	0.0908	0.0998	0.0864	0.0948	0.103	0.115	0.11	0.0947	0.0891	0.1
		Horizontal Differentiation Coefficients															
Parameter		$\alpha_{PM}$	$\alpha_{Agr}$	$\alpha_{Def}$	$\alpha_{Edu}$	$\alpha_{Env}$	$\alpha_{Fin}$	$\alpha_{ForEco}$	$\alpha_{ForRel}$	$\alpha_{GEA}$	$\alpha_{Home}$	$\alpha_{Health}$	$\alpha_{Just}$	$\alpha_{Labour}$	$\alpha_{Plan}$	$\alpha_{SciTech}$	$\alpha_{Trans}$
Est	-	1.22	4.37***	3.49***	-1.26***	-1.57***	1.28***	-0.316	-0.153	0.382	-0.418	-1.5***	1.47***	-3.57***	-0.23	-0.492	-0.976**
	-	0.921	0.556	0.528	0.405	0.383	0.482	0.385	0.43	0.346	0.397	0.449	0.468	0.559	0.32	0.377	0.439

Signif. Codes: \*\*\*: 0.01, \*\*: 0.05, \*: 0.1

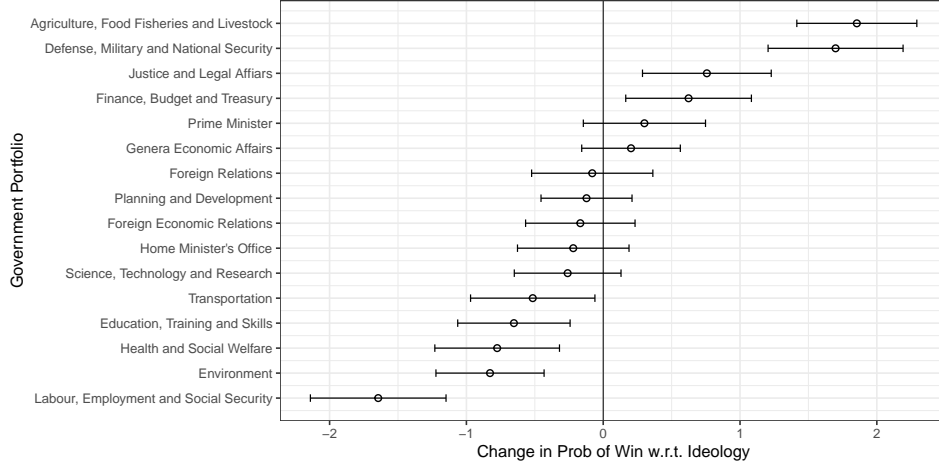
<sup>a</sup> Note: This table reports the estimates and the standard error of the model parameters. Here the vertical differentiation between government portfolios is highlighted by parameters  $\gamma_{PM}$  to  $\gamma_{Trans}$ . The horizontal differentiation between government portfolios is highlighted by  $\alpha_{PM}$  to  $\alpha_{Trans}$ . To calculate standard errors I use the hessian of the log-likelihood and also the observation-wise gradient of the log-likelihood. Here the number of observations is  $N = 356$ , i.e. the number of government cabinets in the data. The loglikelihood value here is given by  $LL = -5864.47$ .

The coefficients on party ideology account for vertical differentiation. Moreover, the nature of vertical differentiation that is shown by these estimates has been documented in the literature. Now I would like to proceed to discuss the estimates of party ideology coefficients. These coefficients account for horizontal differentiation.

The portfolios corresponding to Agriculture, Defense, Finance and Justice are sought by the right more than the left. This is evident as the coefficients corresponding to party ideology for these portfolios are given by 4.37 (0.556), 3.49 (0.528), 1.28 (0.482), and 1.47 (0.468). Each of these estimates are significant at 1% level of significance.

The portfolios corresponding to Labour, Environment, Education, Health, and Transportation are sought more by the left than the right. This is evident as the coefficients corresponding to party ideology for these portfolios are given by -3.57 (0.559), -1.57 (0.383), -1.26 (0.405), -1.5 (0.449), and -0.976 (0.439). Each of these estimates are significant at 1% level of significance.

I also analyze the change in equilibrium allocation probability of a cabinet seat,  $k$ , with respect to change in party ideology. This is given by the average of the numerical derivative of the allocation probability with respect to ideology due to the change in party ideology. Since there is a discontinuous function involved I can not directly utilize analytic derivatives here. This quantity can also be termed as the average marginal effect of equilibrium allocation probability with respect to ideology and it is given as followed:



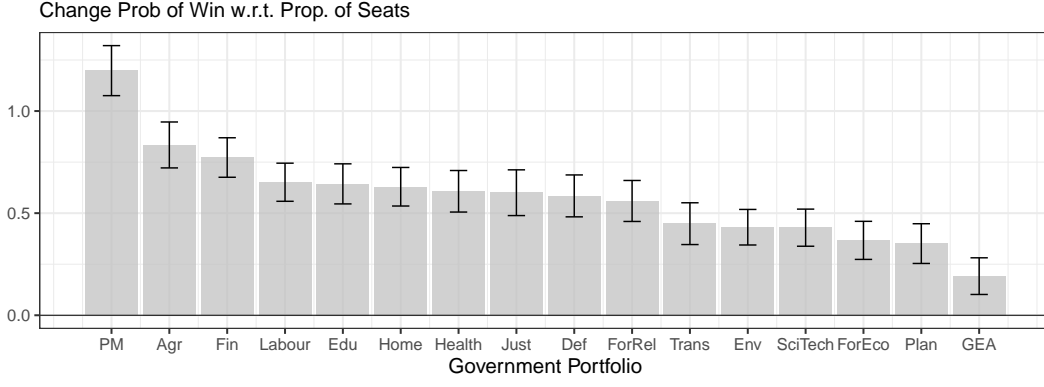
**Figure 3:** This figure plots the average marginal change in equilibrium allocation probability due to small change in party ideology. I use numerical differentiation to calculate these quantities. Standard Errors use Delta Method. Portfolios such as agriculture, Defense, Finance and Justice exhibit significant increases in equilibrium allocation probability if a party becomes more 'right'. The opposite holds for Labour, Environment, Education, Health, and Transportation. Lastly, no such pattern holds for PM, Foreign Economic Affairs, Foreign Relations, General Economic Affairs, Home Affairs and Planning. These effects take account of the strategic interaction across players and therefore are causal.

$$\nabla_{\text{Ideology}}^k \text{Alloc. Prob.} = \frac{1}{C} \sum_{c=1}^C \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{h} \left[ \begin{array}{l} \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k(I_i + h) - \gamma_k P_i\}}{\exp \{-\alpha_k(I_i + h) - \gamma_k P_i\} + \sum_{j \neq i} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \\ - \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \end{array} \right] \quad (6.1)$$

here  $h = 10^{-8}$ .

The results from this exercise is given in the Figure 3. Here I have evaluated the standard errors using delta method where the Jacobian is computed using numerical differentiation as well. Clearly from the marginal effects, if a party switches from left to right then there is a steep increase in probability of allocation for portfolios such as Agriculture, Defense, Finance and Justice. These increases are significant at 1% level of significance. At the same time there is also a steep decline in probability of allocation for portfolios such as Labour, Environment, Education, Health, and Transportation.

The portfolios where horizontal differentiation is non-existence there is no significant change in allocation probability. These portfolios are PM, Foreign Economic Affairs, Foreign Relations, General Economic Affairs, Home Affairs and Planning. These are equally sought by candidate irrespective of their ideology. Therefore the marginal



**Figure 4:** This figure plots the average marginal change in equilibrium allocation probability due to small change in proportion of seats of a party. I use numerical differentiation to calculate these quantities. Standard Errors use Delta Method. Clearly, PM seats has the highest increment in allocation probability than other portfolios. The changes for portfolios such as Agriculture, Finance, Labour, and Education are not statistically indistinguishable from each other. This indicates that they have similar ranks in terms of their vertical differentiation for each party. These effects take account of the strategic interaction across players and therefore are causal.

effects of ideology on equilibrium allocation probability for these portfolios is insignificant.

I also calculate the same quantities with respect to changes in cabinet seat size. These quantities represent the change in equilibrium allocation probability due a change in proportion of seats a party has in the government. These capture the extent of portfolio surplus extraction that can be attributed to the bargaining weight the party has. Formally these are defined as:

$$\nabla_{\text{Prop. of Seats}}^k \text{Alloc. Prob.} = \frac{1}{C} \sum_{c=1}^C \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{h} \left[ \begin{array}{l} \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k (P_i + h)\}}{\exp \{-\alpha_k I_i - \gamma_k (P_i + h)\} + \sum_{j \neq i} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \\ - \max \left\{ 0, 1 - \frac{(N_c - 1) \exp \{-\alpha_k I_i - \gamma_k P_i\}}{\sum_{j=1}^{N_c} \exp \{-\alpha_k I_j - \gamma_k P_j\}} \right\} \end{array} \right] \quad (6.2)$$

here  $h = 10^{-8}$ .

The results from calculating these quantities is given in Figure 4. Clearly, PM seats has the highest increment in allocation probability than other portfolios. This disproportionate jump in the equilibrium allocation probability speaks to the results obtained in Fujiwara and Sanz (2020). The changes for portfolios such as Agriculture, Finance, Labour, and Education are not statistically indistinguishable from each other.

This indicates that they have similar ranks in terms of their vertical differentiation for a centered party.

The least valued government portfolios are given by Science Technology, Foreign Economic Relations and General Economic Affairs. These portfolios are also often associated with least amount of funds and executive power in terms of forming regulations.

## 7 Counterfactual Experiments

### 7.1 Proportion of Allocations Explained by Horizontal Differentiation

In this section I estimate how many government portfolio allocations are explained by Horizontal Differentiation. I do this exercise for each government portfolio. In order to estimate this proportion, I calculate the predicted portfolio allocation for "No Horizontal Differentiation" and "Baseline Model". Then I calculate the proportion of cabinets in which the two cases do not agree with each other. This proportion provides me with the estimate for the proportion of government portfolio allocations that can be explained by horizontal differentiation.

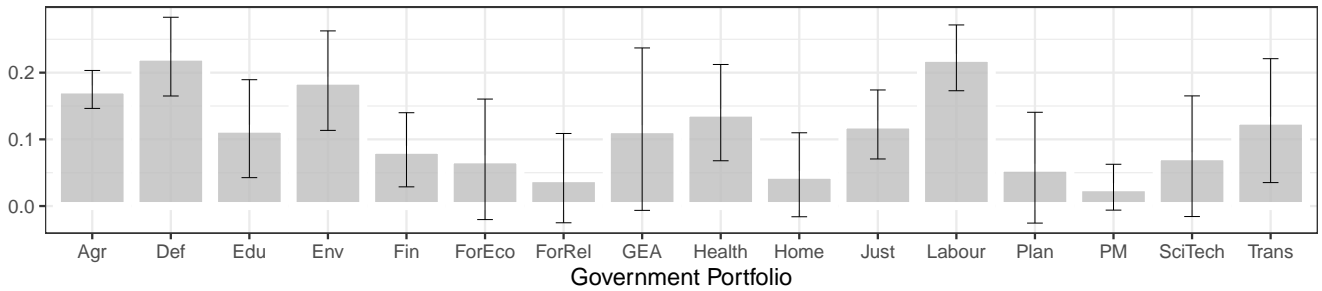
Moreover, I draw a large sample of potential parameter values from the asymptotic distribution of the parameter estimates. I calculate the above differences for each drawn parameter value. The standard error of the obtained distribution provides me with the standard error of the counterfactual estimate. I use this standard error to statistically infer if these proportions are significant or not.

The results from this exercise are given in Figure 5 and Table 4. Note that government portfolios such as Defense, Labour, Agriculture, Environment, Finance, Education, Justice, and Transportation can not be fully explained by just vertical differentiation across these portfolios alone. The proportion of allocations that can be explained by horizontal differentiation is highest for Defense at 22.4% followed by Labour at 22.4%, this goes all the way till Finance where 8.4% of the allocations can be explained by horizontal differentiation.

Allocations of portfolios such as PM, Science and Technology, Foreign Relations, General Economic Affairs, and Home Affairs can be explained even if horizontal dif-



Proportion of Allocations Explained by Horizontal Differentiation



**Figure 5:** This figure shows the estimated proportion of allocations that can be explained by horizontal differentiation. Here I consider to set of simulations, the first one considers the baseline model— that allows for horizontal differentiation— and the second considers the modified model where the horizontal differentiation channel is shut down. The differences between these two scenarios forms the basis for my estimates.

**Table 4:** Proportion of Allocations Explained by Horizontal Differentiation

	PM	Agr	Def	Edu	Env	Fin	ForEco	ForRel	GEA	Home	Health	Just	Labour	Plan	SciTech	Trans
Estimate	0.0283	0.1748	0.2240	0.1160	0.1880	0.0844	0.0701	0.0419	0.1153	0.0469	0.1401	0.1223	0.2222	0.0575	0.0747	0.1281
	0.018	0.015	0.030	0.037	0.038	0.028	0.046	0.034	0.062	0.032	0.037	0.026	0.025	0.042	0.046	0.047

<sup>a</sup> This table shows the estimated proportion of allocations that can be explained by horizontal differentiation. Here I consider to set of simulations, the first one considers the baseline model— that allows for horizontal differentiation— and the second considers the modified model where the horizontal differentiation channel is shut down. The differences between these two scenarios forms the basis for my estimates.

ferentiation is shut down. This shows that horizontal differentiation of government portfolios can be critical in explaining some of the portfolio allocations however for the others it may not play any role. This is true for the ones where vertical differentiation dominates the preferences of the parties.

Horizontal differentiation based on ideology is similar to what [Bäck et al. \(2011\)](#) study. The authors uncover dependence of cabinet allocations on party election manifestos and test multiple hypothesis. Here I micro-find this pattern by addressing the strategic dependence across candidates and provide causal dependence of allocations on party ideology.

## 7.2 Pareto Optimal Allocations

In this section I calculate the first best government portfolio allocations. For this purpose I find the allocation that a social planner will propose. I consider two types of social welfare functions. In the first social welfare function I assume that each party has equal weight in the social planner’s problem. The problem is given by the follow-

Table 5: Counterfactual Strategic vs Pareto Optimal

	Strategic	Pareto Optimal	Difference
Social Welfare Function 1	8.780	23.484	14.703***
	1.063	2.850	1.794
Social Welfare Function 2	5.436	7.811	2.376***
	0.535	0.932	0.417
<i>Expected Delay</i>			
Average willingness to wait	0.4281	1.06e-09	-0.4281***
	0.0504	5.22e-12	0.0504
Max willingness to wait	5.9959***	9.19e-09	-5.9959***
	0.3546	1.15e-10	1.7172

*Signif. Codes: \*\*\*: 0.01, \*\*: 0.05, \*: 0.1*

<sup>a</sup> *Note: This table reports the estimates and the standard error of the counterfactual experiment that compares Pareto Optimal Allocation of Portfolios with that of the Strategic Allocation of Portfolios. I find considerable room for improvement within the Two settings. This improvement is driven due to practically 100% reduction in government formation delay.*

ing:

$$\begin{aligned}
 \text{Social Planner Problem 1} &= \max_{\{T_{1k}, \dots, T_{Nck}\}_{k=1}^K} SWF_1 \left( \{T_{1k}, \dots, T_{Nck}\}_{k=1}^K ; \{I_i, P_{ic}\}_{i=1}^{N_c} \right) \\
 &= \max_{\{T_{1k}, \dots, T_{Nck}\}_{k=1}^K} \sum_{k=1}^K \left( \sum_{i=1}^{N_c} \exp\{\alpha_k I_i\} \frac{T_{ik}}{\sum_j T_{jk}} - \exp\{-\gamma_k P_{ic}\} T_{ik} \right)
 \end{aligned} \tag{7.1}$$

In this problem the social planner can choose very small bids for each party making the role of proportion of seats negligible. In order to bypass this issue, I consider the second type of problem by the social planner. In this problem the social planner weights the utility of each party  $i$  by  $P_{ic}$ . The proportion of seats party  $i$  has in the cabinet. This

problem is given by:

$$\begin{aligned}
\text{Social Planner Problem 2} &= \max_{\{T_{1k}, \dots, T_{Nck}\}_{k=1}^K} SWF_2 \left( \{T_{1k}, \dots, T_{Nck}\}_{k=1}^K ; \{I_i, P_{ic}\}_{i=1}^{N_c} \right) \\
&= \max_{\{T_{1k}, \dots, T_{Nck}\}_{k=1}^K} \sum_{k=1}^K \left\{ \sum_{i=1}^{N_c} P_{ic} \left( \exp\{\alpha_k I_i\} \frac{T_{ik}}{\sum_j T_{jk}} - \exp\{-\gamma_k P_{ic}\} T_{ik} \right) \right\}
\end{aligned} \tag{7.2}$$

The rationale for weighting the utility of the party within the ruling cabinet is to respect the electorate's mandate. Each party represents the preference of voters. These proportion of voters for each preference is given by the proportion of seats each party has within the ruling cabinet.

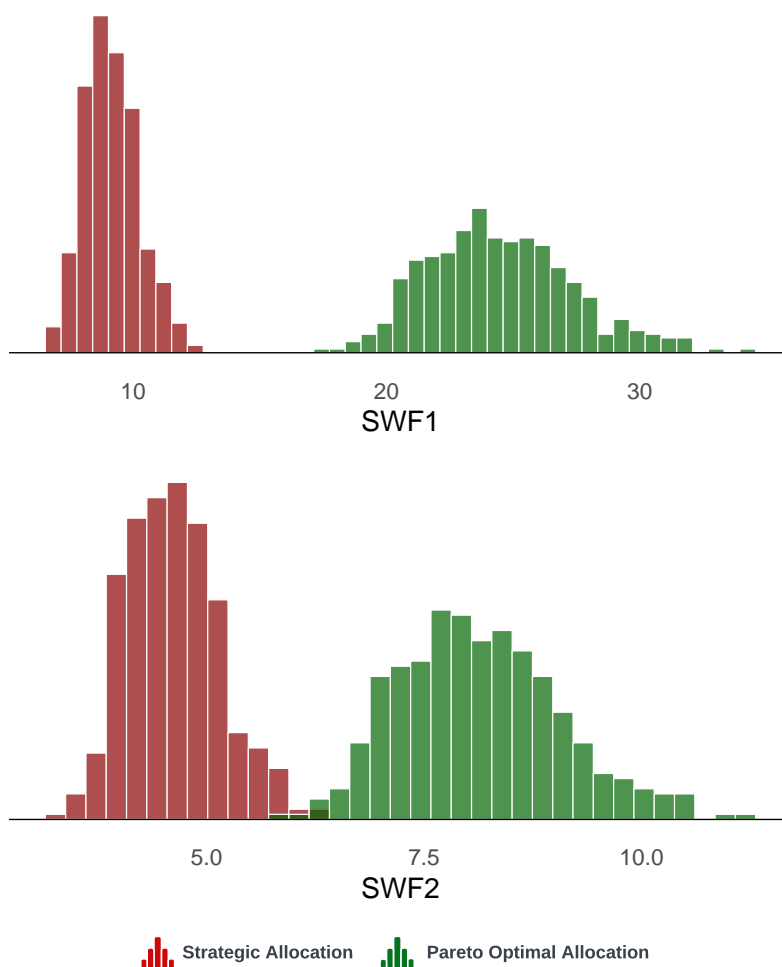
In order to infer the improvement made by the first best allocation I carry out the following set of steps. First I estimate the difference in social welfare values in the strategic setting and also the pareto optimal setting. Then I estimate the distribution of the average social welfare. For this I calculate the average across all cabinets for each parameter values drawn from the asymptotic distribution of the estimates. I use the distribution for calculating the standard errors of these counterfactual estimates. I do these steps for both the definitions of the social welfare function.

In Table 5 I show the social welfare values under the case of the strategic model, Pareto Optimal allocation and also the difference between the two. If we rely on social welfare function 1, it predicts that there is roughly a 200% improvement, which is significant at 1% level of significance. Clearly, such an improvement is unrealistic as this welfare function puts the same weight on each party irrespective of their contribution made in the government formation.

I also show the social welfare values under the case of the strategic model, Pareto Optimal allocation and the difference between the two according when using the SWF2 function. Here the counterfactual predicts that there will be roughly a 50% improvement if one uses a mechanism, that can implement the first best allocation, to allocate government portfolios across. The model predicts that this improvement will be driven by the reduction in delay times and costs.

I also show the estimated distribution across parameter draws in Figure 6. Clearly the shift in the predicted welfare under SWF1 is much higher for SWF1. However, the jump is much more realistic for the case of SWF2.

## Estimated Welfare Distribution



**Figure 6:** *This figure plots the estimated average welfare distribution obtained across 400 parameter draws from the estimated asymptotic distribution of parameter estimates. Here I compare aggregate welfare under the baseline Colonel Blotto game that approximates the multi-dimensional bargaining protocol with the pareto optimal allocation of the same game. There is a 100% increase in average welfare of a cabinet by employing the optimal allocation. This can be achieved by using various mechanisms such as VCG.*

## 8 Conclusion

In this paper, I provide one of the first pieces of evidence that support the claim that political parties are heterogeneous not only on the legislative side but also on the executive side of government. I show that parties horizontally differentiate government portfolios, and their ideology strongly predicts these preferences. I show this by providing reduced-form evidence that analyses the correlation of portfolio allocation with the party ideology. These correlations do not account for strategic interactions, so I

construct a model that allows for heterogeneous preferences and possesses a potential for estimation.

I find that the horizontal differentiation of portfolios can explain a significant proportion of portfolio allocations by political parties. Previous researchers have not accounted for such differences, so it is unclear how many prior results are robust to this preference heterogeneity. Moreover, the data rejects the traditional assumption of only one pie equally valued by all— the core of the models. This assumption also rules out Pareto improvements as there is no room for improving allocations. The setup I provide in this model shows that substantial gains can be obtained from obtaining a better allocation of government portfolios across parties than in the Nash Equilibria.

The paper also opens room for further research. The first is to investigate determinants that determine the heterogeneous preferences that we have obtained. The paper still does not answer why the left presses more for the Labour portfolio and not the Defense. The second motivates the need to expand the current bargaining models that bargain over multiple issues such that parties have distinct preferences over these issues. A situation that predominantly exists in today's time, yet we lack the theoretical tools to analyze these strategic interactions.

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# A Additional Tables and Figures

Table 6: Robustness Budget Constraints

		Vertical Differentiation Coefficients															
Parameter	$\sigma$	$\gamma_{PM}$	$\gamma_{Agr}$	$\gamma_{Def}$	$\gamma_{Edu}$	$\gamma_{Env}$	$\gamma_{Fin}$	$\gamma_{ForEco}$	$\gamma_{ForRel}$	$\gamma_{GEA}$	$\gamma_{Home}$	$\gamma_{Health}$	$\gamma_{Just}$	$\gamma_{Labour}$	$\gamma_{Plan}$	$\gamma_{SciTech}$	$\gamma_{Trans}$
Estimate	0.323	5.51***	2.9***	2.07***	2.1***	1.58***	2.43***	1.48***	1.95***	1.15***	2.07***	1.93***	2.05***	2.05***	1.45***	1.61***	1.65***
	0.00383	0.544	0.0291	0.0519	0.144	0.145	0.0487	0.198	0.106	0.307	0.136	0.146	0.128	0.103	0.12	0.116	0.104

		Horizontal Differentiation Coefficients															
Parameter	$\delta$	$\alpha_{PM}$	$\alpha_{Agr}$	$\alpha_{Def}$	$\alpha_{Edu}$	$\alpha_{Env}$	$\alpha_{Fin}$	$\alpha_{ForEco}$	$\alpha_{ForRel}$	$\alpha_{GEA}$	$\alpha_{Home}$	$\alpha_{Health}$	$\alpha_{Just}$	$\alpha_{Labour}$	$\alpha_{Plan}$	$\alpha_{SciTech}$	$\alpha_{Trans}$
Estimate	0.0633	1.71	3.19***	2.05***	-1.01	-1.62***	0.635***	-1.08	-0.513	-0.92	-0.572	-1.22	0.433	-1.69***	-1.03	-0.987	-1.26**
	0.106	1.09	0.35	0.059	0.936	0.464	0.0662	2.01	1.35	3.2	1.11	1.72	0.614	0.486	0.82	1.55	0.621

*This table reports the estimates and the standard error of the model parameters. Here the vertical differentiation between government portfolios is highlighted by parameters  $\gamma_{PM}$  to  $\gamma_{Trans}$ . The horizontal differentiation between government portfolios is highlighted by  $\alpha_{PM}$  to  $\alpha_{Trans}$ . To calculate standard errors I use the hessian of the log-likelihood and also the observation-wise gradient of the log-likelihood. Here the number of observations is  $N = 356$ , i.e. the number of government cabinets in the data. The loglikelihood value here is given by  $LL = -5864.47$ .*

## B Additional Figures

Figure 7: Net Value of Government Portfolio

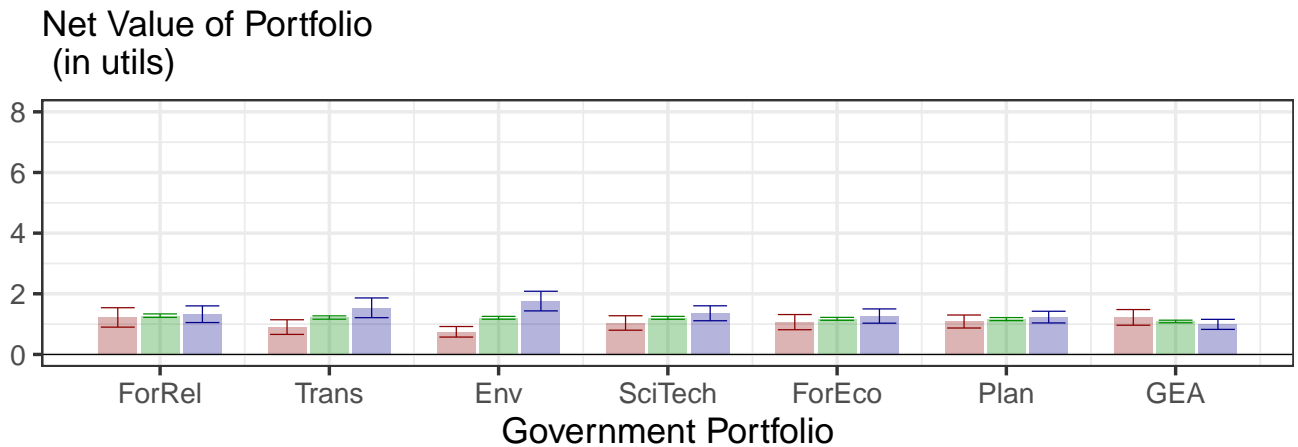
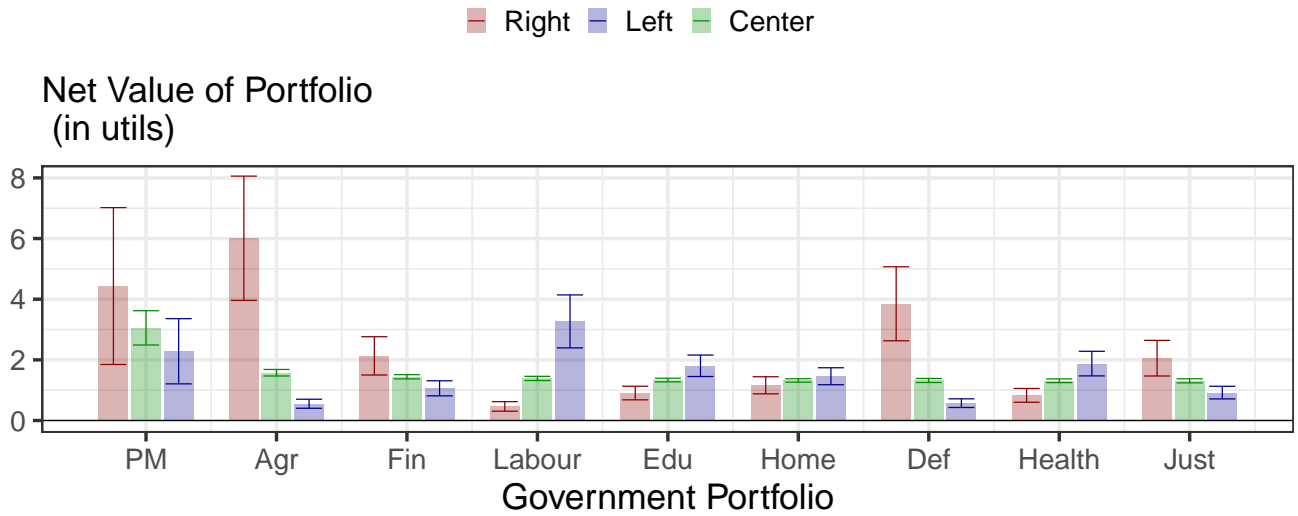


Figure 8: The figure calculates net utility parties obtain from a government portfolios for a center party (median of observed ideology distribution), a left party (33rd percentile of ideology distribution) and a right party (67th percentile of the observed ideology distribution). Note how Agriculture is highly valued by a right party but not at all by a left party. An opposite pattern holds for the Labour department. The standard errors are calculated using delta method.

Figure 9: Average Government Portfolio Rankings by Ideology

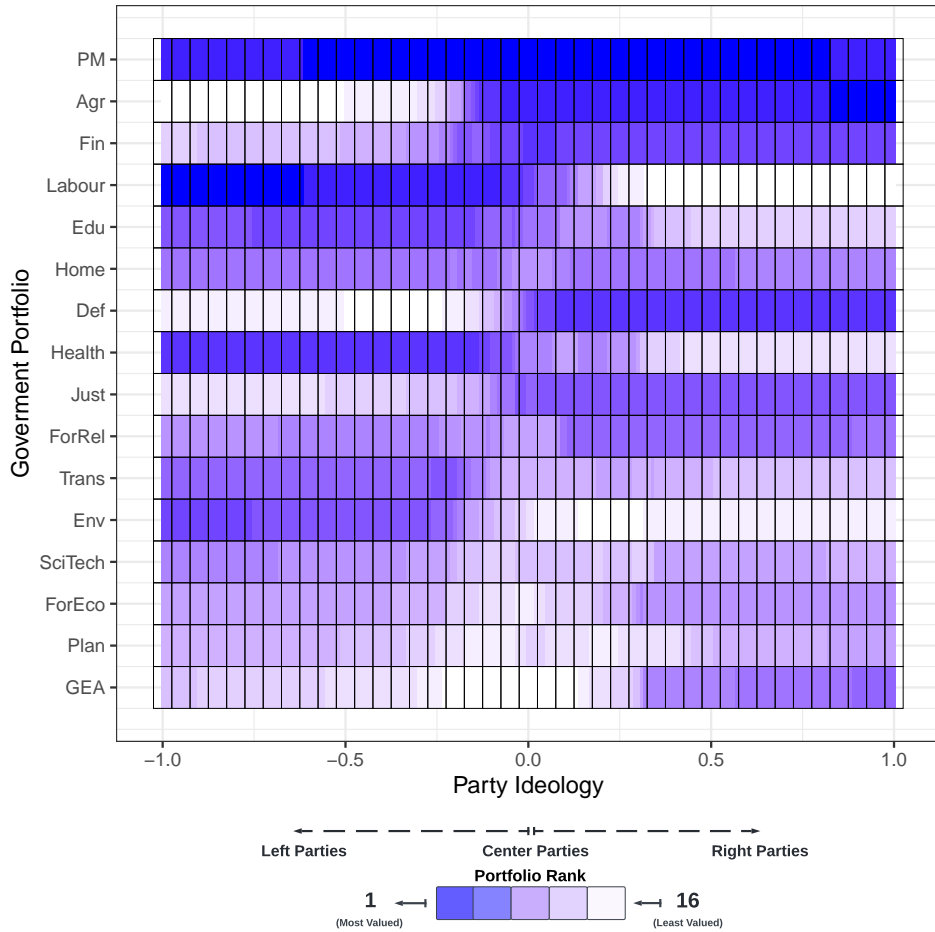


Figure 10: The figure calculate ranking of government portfolios for a grid of ideology values. The objective of the figure is to show the flexibility the empirical specification possess. Here note the amount of non-linear rankings that specification can support despite having only two parameters per portfolio. The darker blue colors are the highest ranked portfolio and the lighter colored are lower ranked portfolios.