# Rallies and Popularity: The Case of Indian Parliamentary Elections

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**Abstract**

The paper constructs a model of dynamic electoral competition where politicians compete against each other to stay popular on election day. The model possesses a finite time horizon with a perfect information structure which results in a unique equilibrium. An extensive simulation study is conducted to understand the model's comparative statics, which provides essential intuition that can be used to explain the dynamics of campaign strategy used in practice. The model is applied to the 2019 Indian General Election to test how fairly it performs. This paper also provides one of the first examinations of Modi rallies that helps us to get a sense of how effective Modi rallies were in the 2019 Indian General Elections.

## 1 Introduction

Over the years political rallies have become a large part of electoral campaigns. They are carried out in many democracies in developing and developed nations alike. For instance, in South Asia, a rally in Kolkata had half a million attendees (Al Jazeera, 2019). In Tanzania, rallies are a more commonly used campaigning instrument than canvassing (Paget, 2019).

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The model in this paper precedes the model used in "Rally the Vote: Electoral Competition with Direct Campaign Communication"

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In Latin America, specifically Ecuador and Argentina, rallies form essential features of campaigns (De la Torre and Conaghan, 2009; Szwarcberg, 2012). In the U.S. Donald Trump's rallies had an average attendance of 5,505[1] during the 2016 fall campaign. Nine of these rallies had more than 10,000 attendees. In the fall campaigns of 2012 and 2016, political rallies constituted 44.5%[2] of all campaign activities involving presidential candidates. Fundraisers followed it at 17.4%.

Even though rallies are a favored campaigning instrument, we know very little about them. This lack of work on political rallies dramatically contrasts with the work on the efficacy of political advertising (Gordon and Hartmann, 2013; Hill et al., 2013; Gerber et al., 2011; Spenkuch et al., 2018), strategic allocations of advertising (Erikson and Palfrey, 2000; Gordon and Hartmann, 2016; Snyder, 1989), and also dynamic inter and intra-electoral spending (Acharya et al., 2022; de Roos and Sarafidis, 2018; Kawai and Sunada, 2022). Empirical work on political rallies has proven challenging due to endogenous rally decisions, measurement error, candidate level heterogeneity, and a low number of observations. These concerns are hard to address[3]. Theoretical and structural work faces challenges imposed by multiple equilibria, which arise due to a finite time horizon in these settings.

Studies in political science and communication have provided varying estimates over the years. These estimates remain sensitive to the type of study being considered. Studies considering campaign events before 2000, such as Shaw (1999), have found temporary and low effects. At the same time, studies with partial randomization, such as Shaw and Gimpel (2012) have found substantial and significant effects. Despite these studies, estimates have remained under doubt. This unreliability can be broken down into three reasons: 1) Effectiveness of campaign events (or political rallies in our case) should be individual specific. 2) Opponent decisions correlate with candidates' decisions and, therefore, should be considered as a part of the study. 3) These variables are endogenous, i.e., trial heat (popularity in our setting), candidate decision, and opponent decision are equilibrium objects. Due to these three shortcomings, results provided by past scholars can not be interpreted as causal.

We focus on how dynamic uncertainty shapes strategic interactions between two competing electoral campaigns. We study this in the context of dynamic political rally choices. As mentioned before, due to complications created by multiple equilibria in dynamic finite

[1]This figure is calculated using news reports on individual rallies from multiple news providers. Complete details on sources of each Trump rally can be provided upon request.

[2]I used candidate calendars made available by Appleman (2012, 2016) for calculating this figure.

[3]For instance, Shaw and Gimpel (2012) randomized a gubernatorial candidate's visit locations in Texas but not the opponent's visit locations. More recently, Snyder and Yousaf (2020) did an event study at the DMA level by using Cooperative Election Study surveys. The authors find significant effectiveness for Trump rallies but not for others. Due to a low number of respondents in CES surveys at the day×DMA level, their measures of intention to vote for a candidate carry additional noise, thereby increasing the underlying variance which is carried into their estimates. Moreover, SUTVA is harder to maintain at DMA-level analysis as there can be geographical spillovers.

time horizon games, studying the strategic × dynamic behavior of candidates within an election is not always feasible. In our model, a politician may exhibit strategic complementarity and follow his opponent. He can exhibit strategic substitution by avoiding rallying in the exact location of his opponent. He can also exhibit both tendencies[4] throughout the election. Therefore, there is non-trivial dependence between the two candidate's campaigning decisions.

We also take the model to the data to test how fairly it performs in providing a dynamic and cross-sectional fit. For this purpose, we focus on the 2019 Indian Parliamentary Elections. We find encouraging evidence in support of our model by analyzing the model fit. However, due to data restriction, we cannot estimate the model; instead, we consider a grid of possible parameter values and choose the values that provide the best model fit in terms of MSE. We can also explain why INC[5] campaigned more heavily in South Indian states in 2019 than in the North Indian states. The explanation utilizes the intuition that we build in terms of strategic substitution and complementarity.

The paper will first discuss the institutional setting for India and lay out how the phased elections take form. Indian elections take place in a phased manner. India is large country with a huge voting age population. Due to this holding a democratic election is a logistic challenge. Here I discuss how the 2019 elections took place.

Analyzing dynamic games possessing a finite time horizon is a challenging task. One vital issue they possess is the existence of multiple equilibria[6]. So far, the majority of dynamic campaigning models have considered inter-electoral (or an infinite horizon) settings (Kawai and Sunada, 2022; Polborn and Yi, 2006; Gul and Pesendorfer, 2012) rather than intra-electoral settings. The models that consider campaigning within an election have to settle for a unique normalization over equilibrium strategies (Acharya et al., 2022), as a unique equilibrium is harder to support. As a result, they still lack prediction on candidate-specific strategies. To this end I construct a dynamic game of perfect information that allows for unique equilibrium and therefore makes it feasible for us to investigate strategic incentive that arise in dynamic electoral competitions.

In this game, office-seeking candidates are subject to electoral competition while facing regional/cross-sectional differences and dynamic uncertainty. Dynamic uncertainty is essential as it accommodates unforeseen circumstances in electoral races that lead to a candidate jumping ahead or falling behind his opponent. Cross-sectional differences address state-specific factors such as a state's natural inclination towards a party or a regional popularity shock. In this model, a candidate's objective is to stay popular in as many states as

[4]Both tendencies can be exhibited by the same politician. In one period, a politician can follow his opponent, and in another period, the politician stays away from his opponent.

[5]Inian National Congress

[6]For instance, Watanabe and Yamashita (2017) showed that to obtain a tractable set of Markov perfect equilibria, one requires strict assumptions that are not feasible for a flexible empirical approach.

possible on election day, and the locally popular candidate receives a pay-off proportional to the state's parliamentary seats.

Candidates in the model must pay a cost to hold a rally. I allow the costs to vary across candidates and states, thereby addressing candidate and state-level heterogeneity. In order to support the perfect information structure of the game, candidates move in a stochastic order. Both candidates are equally likely to be the first or second mover to ensure a candidate does not have an ex-ante first or second mover advantage. Moreover, this ensures that the choice probabilities are uniquely solvable using backward induction.

The model provides comparative statics and the intuition for identification of model parameters. For instance, candidates hold more frequent rallies over time with higher persistence. The intuition behind this comes from the induced lower decay rate that allows campaigning effects to last longer (Hill et al., 2013; Gerber et al., 2011; Acharya et al., 2022). The lower decay incentivizes candidates to hold a higher number of initial rallies. An increase in the cost of rallies introduces a level shift in the probability of rallying. An increase in effectiveness parameter exhibits a non-linear change in campaigning strategy, where rallies increase for smaller effectiveness values and decreases for more significant values. This happens because higher effectiveness enables a candidate to maintain a sufficient level of popularity with fewer rallies.

I also extend the model to the Indian context and calibrate the model to match certain moments. However, due to not having observations on candidate's actual popularity I do not have strong identification. For this I create a grid of parameter values and choose the one which provides the best model fit. I use the corresponding parameter value to formulate how effective Modi rallies were in the 2019 Indian parliamentary elections.

This paper contributes to the literature on political campaigning (Kawai and Sunada, 2022; Erikson and Palfrey, 2000; de Roos and Sarafidis, 2018; Meirowitz, 2008; Polborn and Yi, 2006; Garcia-Jimeno and Yildirim, 2017; Gul and Pesendorfer, 2012; Strömberg, 2008) by constructing a dynamic framework where candidates choose to rally. Strömberg (2008) study campaign state visits and build a model where candidates allocate time across states. The model is static, has identical strategies and does not incorporate decay. I provide a dynamic model with candidate-specific strategies where campaign effects decay. Acharya et al. (2022) study political spending within an election and identify the perceived decay rate associated with campaigning. However, due to the finite time horizon, the authors characterize optimal spending ratios rather than candidate-specific spending strategies. In my model, I utilize a perfect information structure that allows me to study candidate-specific strategies. Kawai and Sunada (2022) study spending across elections, whereas I study rallying decisions within an election. Erikson and Palfrey (2000) estimate a model of fund-raising and campaigning in U.S. house elections while other papers, Meirowitz (2008); Polborn and Yi (2006) study negative campaigning. Gul and Pesendorfer (2012) construct a model where candidates decide when to stop campaigning. Garcia-Jimeno and Yildirim (2017) study strategic interaction between candidates in bipartisan races and media.

I also contribute to the literature on analyzing stochastic goodwill models in marketing and operations research (Kwon and Zhang, 2015; Grosset and Viscolani, 2004; Marinelli, 2007; Doganoglu and Klapper, 2006; Chintagunta and Vilcassim, 1992). I contribute by extending the stochastic goodwill framework to a dynamic discrete game framework. Traditionally these models have studied dynamic advertising for firms that wish to maintain/increase their goodwill among their consumers. Here, I provide a model where the goodwill can only be binary in a given market. Moreover, there is a period-specific capacity constraint on the level of advertising where a marketeer can only advertise in one market at a given time. The contribution here is the capability of the model to provide predictions on advertising across multiple markets at once. Moreover, the time horizon is finite, and the model appeals to situations where the marketing firm faces a specified product launch deadline.

## 2 Institutional Background

India is a Parliamentary democracy and as a result the prime minister is not directly chosen by voters. The whole of India is geographically divided into 543 parliamentary constituencies. Every parliamentary constituency has a parliamentary seat that will be filled by the representative of that constituency. Every voter casts his vote to elect the representative for his or her constituency. Political parties nominate a single or no candidate for each constituency and voters decide which candidate should represent their constituency. The political party or the political alliance that secures more than 50% parliamentary seats gets to choose the prime minister. The prime ministerial candidate for each political alliance tends to be public information. There are two major political alliances in India. One is the National Democratic Alliance (NDA) which generally includes parties that include centre-right or right ideologically inclined parties. The member parties have consisted of Bhartiya Janta Party (BJP, also the de facto leader of the alliance), Shiv Sena (since the Maharashtra 2019 elections, it has seized to be a member), Janta Dal (seized to be a member since 2014), Shiromani Akali Dal (SAD), etc. The second major alliance (centre-left and left parties), that used to be the most dominant alliance prior to 2014, is the United Progressive Alliance (UP). It is led by Indian National Congress (INC) and has had NCP, CPI(M), RJD, DMK as its members in the past governments.

Since NDA has BJP as its de facto leading party, its prime ministerial candidate is Narendra Modi. Similarly UPA has INC as its leading party, and INC's de facto prime ministerial candidate is Rahul Gandhi. We will be focusing on political rallies conducted by these two politicians when we analyze the rally decisions. Unlike United States, here PM candidates care for every parliamentary seat and therefore they attempt to promote their parties in as

many constituencies as possible. Here candidate's success depends not only on the success of his party but on the success of the whole alliance. Therefore, we consider all parliamentary seats here for defining electoral gains of the PM Candidates.
A key difference between how elections are conducted in India with US and also most of the democracies is that all constituencies do not conduct elections at the same time. Elections in this case are divided into phases. In 2019 there were seven phases in which elections took place. In each phase elections are conducted in group of constituencies. The number of constituencies in each phase do not need to be the same. The reason the elections are happen in this phased manner is that the Election Commission of India can not conduct elections for the whole country at the same time. The population of India is gigantic. As a result a large number of polling booths and election officers are required to conduct fair elections. There are not enough resources to do conduct elections at the same time for each constituency and therefore the phased-procedure is followed. Even though elections happen at separate dates, results are announced at the same date for each constituency.

Table 1: Number of Constituencies and Election Dates for Each Phase

| Phase | Number of Constituencies | Election Date |
|---|---|---|
| Phase I | 91 | 11 April, 2019 |
| Phase II | 97 | 18 April, 2019 |
| Phase III | 115 | 23 April, 2019 |
| Phase IV | 71 | 29 April, 2019 |
| Phase V | 51 | 6 May, 2019 |
| Phase VI | 59 | 12 May, 2019 |
| Phase VII | 59 | 19 May, 2019 |

The information of number of constituencies and election dates of each phase is given in Table 1.

# 3 Model

The model is constructed to analyze the interaction between candidate rally choices and their popularity level. In this model, we have $K$ states, $T+1$ periods and two candidates. We assume one popularity measure per state that holds information on both candidates. This popularity measure can take two values in the set $\{H, L\}$. Here $H$ is interpreted as '$R$ is more popular than $D$' while $L$ is interpreted as '$D$ is more popular than $R$'. The game is played over first $T$ periods and in each period a sequential move stage game is played. In this stage game the order of play among the candidates is random. A candidate at his turn must choose at most one state out of $K$ states to rally. The decision making stops at period $T$ and in period $T+1$ every state chooses the popular candidate as the winner. The winning candidate in a state gains all parliamentary seats associated with the state. His total gain is the sum of all parliamentary seats he won.

### 3.1 Preliminary

There are $K = \{1, 2, \ldots, K\}$[7] states and two candidates $\{R, D\}$ who are competing against each other. There are $1, 2, \ldots, T$ periods and a terminal period $T+1$. Decisions will be made in periods $1, 2, \ldots, T$ and the winner is decided in the terminal period $T+1$. Each state has a popularity measure $p_{kt}$ that denotes the popularity level of $R$ and $D$ in state $k$. It can take only binary values of $\{H, L\}$. Moreover, $p_{kt}$ taking value $H$ indicates that at $t$ $R$ is popular and $D$ is unpopular in state $k$. If $p_{kt}$ takes the value $L$ then $R$ is unpopular and $D$ is popular. If the game ends with a candidate being popular in state $k$, then the popular candidate in state $k$ would win.

Candidates can allocate a unit of perishable indivisible good to at most one state in a given period. The indivisible good will be interpreted as a political rally and the action to allocate the good to a state $k$ will be referred to as rallying in state $k$. Good can not be saved, and has a constant candidate specific cost[8] $c_i$ for candidate $i$.

Let $a_{ikt}$ be a binary variable. Then $a_{ikt}$ takes value 1 if candidate $i$ chose to rally in state $k$ at period $t$ otherwise it takes value 0. Then $P(p_{k,t+1} = H | a_{Rkt}, a_{Dkt}, p_{kt})$ is given by the following markov process:

$$P\Big(p_{k,t+1} = H \Big| a_{Rkt}, a_{Dkt}, p_{kt}\Big) = h\Big(\sum_{i \in \{R,D\}} \alpha_i a_{ik,t} + \rho(2 \times \mathbb{1}\{p_{kt} = H\} - 1)\Big) \tag{1}$$

Here, $\alpha_i$ is defined as candidate $i$'s effectiveness in influencing popularity, where $i \in \{R, D\}$. The parameter $\rho$ captures persistence in popularity and is one of the key parameters of the game that determines the interplay of candidate choices and the popularity level. The function $h(x) = \frac{\exp x}{1+\exp x}$ is the logistic function that allows us to have more flexible transition probabilities despite having lower number of parameters. Say $p_t = (p_{1t}, p_{2t}, \ldots, p_{kt})$ popularity vector at time $t$ and let $p \in \{H, L\}^K$ be an arbitrary popularity vector. Define $a_{it}$ as

[7] We use $K$ to denote the set of states and total number of states as well.

[8] This is the average cost of rallies measured in units of present discounted value of electoral votes. This cost also indicates the number of parliamentary seats a candidate is willing to sacrifice in order to not conduct a rally.

$(a_{i1,t}, a_{i2,t}, \ldots, a_{iK,t})$ for each $i \in \{R, D\}$. We assume, that conditional on $a_{Rt}, a_{Dt}, p_t$ popularity in state $k$ and state $k'$ are uncorrelated, that is the following holds:

$$\begin{aligned}\mathbb{E}\Big[\mathbb{1}\{p_{k,t+1} = p_k\} \times \mathbb{1}\{p_{k',t+1} = p_{k'}\}\Big|a_{Rt}, a_{Dt}, p_t\Big] =&\mathbb{E}\Big[\mathbb{1}\{p_{k,t+1} = p_k\}|a_{Rt}, a_{Dt}, p_t\Big] \\ &\times \mathbb{E}\Big[\mathbb{1}\{p_{k',t+1} = p_{k'}\}|a_{Rt}, a_{Dt}, p_t\Big]\end{aligned} \tag{2}$$

Here, we are assuming that conditional on current popularity across states and decisions made by candidates, popularity across states in the next period are uncorrelated with each other. We are not assuming independence of popularity across states nor are we assuming that popularity is uncorrelated across states. We are saying that if popularity in period $t+1$ is correlated across states, then these correlations can be explained by period $t$ popularity and the decisions made by the candidates. Based on this, we can derive $P\Big(p_{t+1} = p\Big|a_{Rt}, a_{Dt}, p_t\Big)$[9].

$$\begin{aligned}P\Big(p_{t+1} = p\Big|a_{Rt}, a_{Dt}, p_t\Big) &= \prod_{k=\{1,\ldots,K\}} P\Big(p_{kt+1} = p\Big|a_{Rkt}, a_{Dkt}, p_{kt}\Big) \\ &= \prod_{k=\{1,\ldots,K\}} \Big[h\Big(\sum_{i\in\{R,D\}} \alpha_i a_{ik,t} + \rho(2 \times \mathbb{1}\{p_{kt} = H\} - 1)\Big) \times \mathbb{1}\{p_k = H\} \\ &+ \Big(1 - h\Big(\sum_{i\in\{R,D\}} \alpha_i a_{ik,t} + \rho(2 \times \mathbb{1}\{p_{kt} = H\} - 1)\Big)\Big) \times \mathbb{1}\{p_k = L\}\Big]\end{aligned} \tag{3}$$

In the above equations, the first equality follows by independence of $p_{kt+1}$ from $p_{k't+1}$ for $k \neq k'$. The second equality follows by substituting $P(p_{kt+1}|a_{Rkt}, a_{Dkt}, p_{kt})$ from eq. 1. Moreover please note by definition of $P(p_{kt+1} = H|.)$, we only need $k^{th}$ components of $a_{Rt}$, $a_{Dt}$ and $p_t$. Now we define electoral pay-offs for candidate $R$. Every state $k$ has a number of parliamentary seats $E_k$. In period $T+1$ if the game terminates with state $p_{k,T+1} = H$, then all electoral college votes of state $k$ are received by candidate $R$ otherwise $R$ receives none. Therefore, $R$'s total pay-off will be the sum of parliamentary seats he received from each state. Therefore, his electoral pay-off is given by:

$$\begin{aligned} V_{R,T+1}(p_{T+1}) &= \sum_{k=1}^{K} E_k \times \mathbb{1}\Big\{p_{k,T+1} == H\Big\} \\ \Rightarrow \mathbb{E}_{p_{T+1}}\Big[V_{R,T+1}(p_{T+1})\Big|p_T, a_{RKT}, a_{DKT}\Big] &= \sum_{k=1}^{K} E_k \times \mathbb{E}\Big[\mathbb{1}\Big\{p_{k,T+1} == H\Big\}\Big|p_T, a_{RkT}, a_{DkT}\Big]\end{aligned} \tag{4}$$

For candidate $D$ his electoral pay-off is defined as $V_{D,T+1} = \sum_k E_k - V_{R,T+1}$. Therefore, if $R$ looses an electoral college vote, $D$ simultaneously wins that vote. Moreover, by using these

[9]Please note that the following holds

$$\mathbb{E}\Big[\mathbb{1}\{p_{t+1} = p\}\Big|a_{Rt}, a_{Dt}, p_t\Big] = P\Big(p_{t+1} = p\Big|a_{Rt}, a_{Dt}, p_t\Big)$$
$$\mathbb{E}\Big[\mathbb{1}\{p_{kt+1} = p_k\}\Big|a_{Rt}, a_{Dt}, p_t\Big] = P\Big(p_{kt+1} = p_k\Big|a_{Rt}, a_{Dt}, p_t\Big)$$

Since, expectation of an indicator function is the probability of the event it indicates.

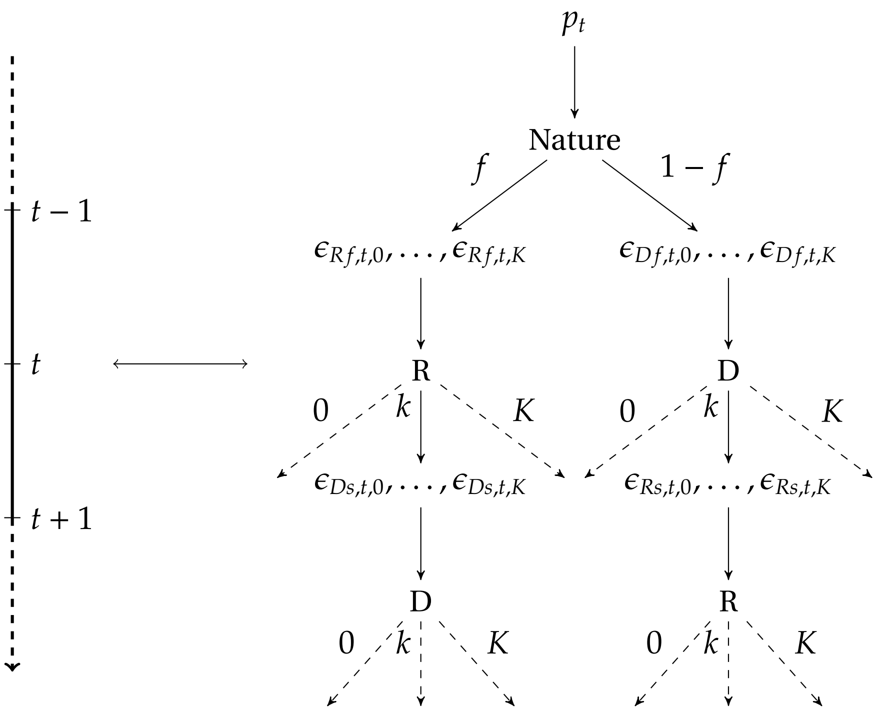



Figure 1: Stage Game

*The stage game for each period $t = 1, 2, \ldots, T$ is provided in this figure. In each period, both candidates observe the popularity level $p_t$. Then nature chooses a first mover and a second mover. Then cost shocks for the first mover are drawn and are observed by both candidates. The first mover decides where to rally. Then cost shocks for the second mover are drawn and observed by both candidates. Second mover makes his decision and then the game proceeds to period $t+1$.*

pay-offs, one can see that $D$'s preferred popularity level is $L$[10]. Therefore, we are enforcing that if $R$ is popular (i.e. $p_{k,T+1} = H$) then $D$ is unpopular and vice-e-versa. Hence, $D$ wishes to keep $R$'s popularity to $L$ (simultaneously his popularity is at $H$) and $R$ wants to keep his popularity at $H$ (simultaneously $D$'s popularity at $L$).

## 3.2 Timing of Decisions and Information

We will denote an arbitrary period as $t$ such that $t \in \{1, 2, \ldots, T, T+1\}$. Game begins at period $t = 1$ and ends at $t = T+1$, where decisions are only made in periods $1, 2, \ldots, T$. A stage game, as described in Figure 1, is played in each period $t \in \{1, 2, \ldots, T\}$. The timing of decisions and revelation of information for the stage game played at period $t$ (refer to Figure 1) will be described in sub-periods $\tau_1, \tau_2, \ldots, \tau_6$. In sub-period $\tau_1$ popularity vector $p_t$ will be observed. In sub-period $\tau_2$, nature will choose a first mover and second mover. In sub-period $\tau_3$, the first mover realizes cost shocks and in sub-period $\tau_4$ he makes his decision.

[10]If at period $T+1$ in state $k$ game ends in state $L$, $R$ will loose all the parliamentary seats from $k$ and simultaneously $D$ will win all parliamentary seats

In sub-period $\tau_5$ second mover realizes his cost shocks and in sub-period $\tau_6$ second mover will make his decision. After this sub-period the game will proceed to period $\tau + 1$. We give more details on each sub-period below.

$\tau_1$ At sub-period $\tau_1$ the popularity level vector $p_t = \left(p_{1t}, p_{2t}, \ldots, p_{Kt}\right)$ is realized and observed by the candidates. Here $p_{kt}$ indicates the current popularity level of the candidates.

$\tau_2$ Nature makes a draw to choose a first mover and a second mover for the stage game. The probability with which $i$ is chosen as the first mover is denoted by $f_i$. Let $f_R = f$ and $f_D = 1 - f$.

$\tau_3$ The first mover $i$ realizes his cost shocks $\epsilon_{if,t} = \left(\epsilon_{if,t,0}, \epsilon_{if,t,1}, \ldots, \epsilon_{if,t,K}\right)$[11] in this sub-period. Here $\epsilon_{if,t,0}$ is the cost shock for not rallying while $\epsilon_{if,t,k}$ is the cost shock for rallying in state $k$. These shocks indicate factors that are not in control of candidates and realized immediately before a decision is made.

$\tau_4$ The first mover ($i$) in period $t$ solves the following bellman equation after observing the current popularity $p_t$ and cost shocks $\epsilon_{ift} = (\epsilon_{if,t,0}, \ldots, \epsilon_{if,t,K})$.:

$$
\begin{aligned}
V_{ift}(p_t, \epsilon_{if,t}) = \max_{k \in \{0,1,\ldots,K\}} \Bigg\{ & - c_i \times \mathbb{1}\{k \neq 0\} + \epsilon_{if,t,k} \\
& + \beta \sum_{k'=0}^{K} \left[ \sum_{p \in \{H,L\}^K} V_{i,t+1}(p) P\left(p \middle| a_{it} = k, a_{jt} = l, p' = p_t\right) \right] \times \sigma_{js,t}\left(l; k, p_t\right) \Bigg\}
\end{aligned} \tag{5}
$$

In order to choose option $k$, $i$ must pay the cost $c_i$ if he chooses to rally, and the random cost shock $\epsilon_{if,t,k}$. These two components form the flow-costs for candidate $i$. Moreover, while making this decision $i$ also takes account of his continuation value associated with each option. The continuation value has the following three components:

- $\sigma_{js,t}(l; k, p_t)$: The probability with which $j$ as $s$ would choose an action given the current popularity level $p_t$ and the action $k$ chosen by $i$ as the first mover.
- $P\left(p \middle| a_{it} = k, a_{jt} = l, p' = p_t\right)$: The probability with which a popularity vector $p$ is realized given the current popularity vector $p_t$, $k$ was chosen by $i$ and $l$ was chosen $j$.
- $V_{i,t+1}(p)$: The value for candidate $i$ in entering period $t + 1$ when the popularity level is $p$. This is candidate $i$'s value function prior to the order of play is realized in period $t + 1$.

[11] These shocks are part of the random utility specification for the candidates. For more details please refer to **?** and **?**

Let $a_{ift}$ be the associated policy function.

$\tau_5$ The second mover $s$ (lets say $i$ was chosen as the second mover) realizes his cost shocks $\epsilon_{is,t} = \Big(\epsilon_{is,t,0}, \epsilon_{is,t,1}, \ldots, \epsilon_{is,t,K}\Big)$.

$\tau_6$ The second mover makes its decision in this period. In addition to observing the popularity level and its cost-shock the second mover also observes the decision made by the first mover. Therefore, $a_{jf,t}$ is also a state variable for the first mover. The second mover solves the following bellman equation after observing $p_t$, first mover action $a_{jft}$ and cost shocks $\epsilon_{ist}$ :

$$\begin{aligned} V_{ist}(a_{jft}, p_t, \epsilon_{is,t}) = \max_{k\in\{0,1,\ldots,K\}} \Bigg\{ & - c_i \times \mathbb{1}\{k \neq 0\} + \epsilon_{is,t,k} \\ & + \beta\Big[ \sum_{p\in\{H,L\}^K} V_{i,t+1}(p) P\Big[p\Big| a_{it} = k, a_{jt} = a_{jft}, p' = p_t\Big]\Big]\Bigg\} \end{aligned} \tag{6}$$

Here $c_i$ and $\epsilon_{is,t,k}$ are the flow-costs from choosing option $k$. The continuation value has two components since the choice made by the opponent is observed. The components are:

- $P\Big[p\Big|a_{it} = k, a_{jt} = a_{jft}, p' = p_t\Big]$: The probability with which a popularity vector $p$ is realized in period $t+1$ given the current popularity vector was $p_t$ and $i$ chose $k$ while $j$ chose $a_{jft}$.
- $V_{it+1}(p)$: The value of entering period $t+1$ at popularity vector $p$.

$a^*_{ist}$ be the associated policy function.

Here we define the value function at popularity vector $p$ prior to order of play in period $t$:

$$V_{i,t}(p_t) = f_i \times \mathbb{E}_{\epsilon_{if,t}}\Big(V_{ift}(p_t, \epsilon_{ift})\Big) + (1 - f_i) \sum_{k=0}^{K} \Bigg[\sigma_{jft}(k; p_t) \times \mathbb{E}_{\epsilon_{is,t}}\Big(V_{ist}(k, p_t, \epsilon_{ist})\Big)\Bigg] \tag{7}$$

The above bellman equation uses the value of being the first mover and the second mover to calculate the value of entering a period when the popularity vector is $p_t$. With probability $f_i$, $i$ is chosen as the first mover and the term $\mathbb{E}_{\epsilon_{if,t}}\Big(V_{ift}(p_t, \epsilon_{ift})\Big)$ is the value of $i$ becoming the first mover before realizing the cost-shocks. Here, the operator $\mathbb{E}_{\epsilon_{if,t}}$ is an expectation operator which calculates the expectation of $V_{ift}(p_t, \epsilon_{ift})$ with respect to the cost shocks $\epsilon_{ift}$. The second term is the expected pay-off of being the second mover. When $i$ is the second mover there are total $K+1$ possibilities that can take place, each possibility corresponds to the decision made by $j$ as the first mover. The expectation is with respect to the conditional choice probability (CCPs from here on) with which $j$ rallies in state $k$. Once an action $k$ is chosen the expected pay-off of being the second mover when action $k$ was chosen by first

mover is given by the term $\mathbb{E}_{\epsilon_{is,t}}\Big(V_{ist}(k, p_t, \epsilon_{ist})\Big)$.
Now with all important ingredients at hand we proceed to characterizing the Subgame Perfect Equilibrium in the next sub-section.

## 3.3 Equilibrium

In this game only one candidate makes their decision at a time. Moreover, at the time of decision making, past actions, popularity levels, and cost shocks are observed and the future actions, popularity levels and cost shocks are not observed. However, candidates are forward looking and have rational expectation of future pay-offs and actions. Therefore, the game at hand is a perfect information game and it can be solved using backward induction by Zermelo's theorem[12].
In a given period current popularity level is pay-off relevant but past popularity is not. Moreover, by Assumption 3.1 we ensure that past cost-shocks are not pay-off relevant either and only the current period cost-shocks are. Therefore, best responses depend on only current popularity, cost shocks and first mover decision (for the second mover).

**Assumption 3.1 (Independent Cost Shocks)** *The cost shocks are independent across all information nodes and actions. That is the following holds:*

$$\epsilon_{im,t,k}\Big|p_t \perp \epsilon_{i'm',t',k'}\Big|p'_{t'} \quad \forall \ (i, m, t, k, p_t) \neq (i', m', t', k', p'_{t'}) \tag{8}$$

In Assumption 3.2, we further assume that cost shocks are drawn from a continuous distribution, in fact from type-1 extreme value distribution. By using a continuous distribution we are able to ensure that a unique Subgame Perfect Equilibrium exists almost surely. Since only one candidate makes a decision at a given time we do not need to search for any fixed point in order to find an equilibrium. Therefore, the only way multiple equilibria would exist is when a candidate is indifferent between two actions. However, since the cost shocks are drawn from a continuous distribution, indifference occurs with probability zero. Another advantage of using type-1 extreme value distribution is to have lower computation burden while evaluating CCPs. Moreover, the use of this distribution is very common in literature that relies on using Discrete Choice models, Dynamic Discrete Games and etc.

**Assumption 3.2 (Distribution of Cost Shocks)** *Cost shocks are drawn from Type-1-Extreme-Value distribution:*

$$\epsilon_{im,t,k}|p_t \sim T1EV \quad \forall \ (i, m, t, k, p_t) \tag{9}$$

[12]Please refer to Proposition 9.B.1 for Zermelo's theorem and Proposition 9.B.2 for existence of pure strategy subgame perfect Nash equilibrium in finite time games of perfect information **?**

Based on our discussion so far and the two assumptions we can show that best responses can defined as a function of current popularity, cost shocks and, in the case of the second mover, first mover action. Below, we characterize the equilibrium conditional choice probabilities. The Proposition 3.1 lays out this characterization. If one knows electoral pay-offs then it is possible to evaluate all period specific value functions and CCPs of both candidates at each possible level of popularity. Eq. 4 defines electoral pay-offs and by using this equation, Proposition 3.1 uses backward induction to characterize period-specific value functions and CCPs.

**Proposition 3.1 ( Value Functions and CCPs)** *Given eq. 4, which defines electoral pay-off $V_{i,T+1}$, the following holds for all $t = 1, 2, ..., T$*

- *The value function $V_{i,t}$ takes the following functional form:*

$$V_{i,t}(p_t) = f_i \times \ln\left( \sum_{k=0}^{K} \exp\left\{u_{if,t}(k;p_t)\right\}\right) + (1-f_i) \times \sum_{k=0}^{K} \left[\sigma_{jf,t}(k;p_t) \ln\left( \sum_{l=0}^{K} \exp\left\{u_{is,t}(l;k,p_t))\right\}\right)\right] \tag{10}$$

- *The expected probability of $i$ choosing action $k$ as the first mover is given by:*

$$\sigma_{if,t}(k;p_t) = P\Big(k = a^*_{if,t}(p_t, \epsilon_{if,t})\Big) = \frac{\exp\Big(u_{if,t}(k;p_t) - u_{if,t}(0;p_t)\Big)}{1 + \sum_{l=1}^{K} \exp\Big(u_{if,t}(l;p_t) - u_{if,t}(0;p_t)\Big)} \tag{11}$$

- *The probability of $i$ choosing action $k$ as the second mover is given by:*

$$\sigma_{is,t}(k;l,p_t) = P\Big(k = a^*_{is,t}(a_{jft} = l, p_t, \epsilon_{is,t})\Big) = \frac{\exp\Big(u_{is,t}(k;l,p_t) - u_{is,t}(0;l,p_t)\Big)}{1 + \sum_{q=1}^{K} \exp\Big(u_{is,t}(q;l,p_t) - u_{is,t}(0;l,p_t)\Big)} \tag{12}$$

  *Where, the option specific value function, $u_{if,t}(k;p_t)$, for $i$ when he is the first mover at period $t$ at popularity level $p_t$ satisfies the following:*

$$u_{if,t}(k;p_t) = \sum_{l=0}^{K} u_{is,t}(k;l,p_t) \times \sigma_{js,t}(l;k,p_t) \tag{13}$$

  *The option specific value function, $u_{is,t}(k;p_t)$, for $i$ when he is the second mover at period $t$ at popularity level $p_t$ and the first mover chose $l$ satisfies the following eq. 14:*

$$u_{is,t}(k;l,p_t) = -\,c_i \times \mathbb{1}\{k \neq 0\} + \beta \sum_{p \in \{H,L\}^K} V_{i,t+1}(p) P\Big[p \Big| a_{it} = k, a_{jt} = l, p' = p_t\Big] \tag{14}$$

The proposition recursively characterizes the equilibrium choice probabilities and value functions for a candidate $i$. The option specific value function of the first mover and the second mover governs the choice probabilities. Second mover's option specific value function solely depends on the expectation of value function in the next period with respect to the transition probability. Due to the nature of transition probabilities at a given time, a second mover has more incentive to rally in a state where the first mover rallied and the current popularity in the state is in second mover's favor. However, if the current popularity in a state $k$ is not in second mover's favor and his opponent rallied, then he has lower incentive to rally in that state. Due to the binary structure of popularity both candidates exhibit opposing tendencies as second mover. That is, if $R$ considers rallies as strategic complements in state $k$ then $D$ would consider rallies as strategic substitutes. The opposite happens if popularity was $L$ in state $k$. With multiple states and a candidate would find himself to be popular in some of them and unpopular in the other can have opposing forces to the candidate himself. In some states he exhibits strategic complementarity and in others he would exhibit strategic substitution. We discuss more on this in Section 3.5.
The first mover option specific value function $u_{if,t}(k;p_t)$ is governed by both effects at the same time. This happens because option specific value function of a candidate $i$ as a first mover depends on his $u_{is,t}(k;p_t)$ and $\sigma_{js,t}(.,k,p_t)$. If $i$ considers rallying in state $k$ as strategic complements as a second mover then his opponent considers it as a strategic substitute. Therefore, the $u_{if,t}(k,p_t)$ has a strategic complement and strategic substitute part and hence the first mover's behavior depends on which of the two dominates.

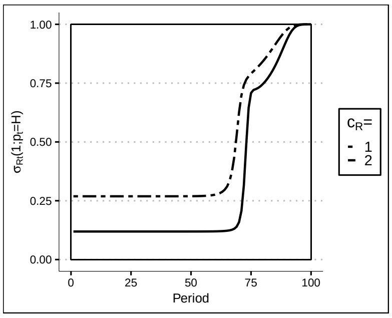
## 3.4 Comparative Statics

Analyzing first mover and second mover probabilities separately makes results more complicated. Second mover probabilities depend on popularity and the first mover actions. For every popularity and first mover action pair we will have a separate conditional choice probability. Therefore, to keep our analysis simple, we compute CCPs prior to the order of play. By doing so, we only need to analyze CCPs conditional on popularity level of $R$. In order to do so first we compute probability of observing an rally choice profile in Proposition 3.2.

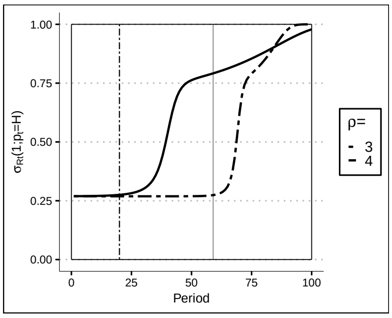
**Proposition 3.2 (Popularity-to-Rally Probabilities)** *The probability of observing $a_t = (a_{Rt}, a_{Dt})$ by $R$ and $D$ when $R$'s popularity vector is $p_t$ at period $t$ is given by:*

$$\underbrace{\sigma_t(a_{Rt}, a_{Dt}; p_t)}_{P[a_{Rt}, a_{Dt}|p_t, t]} = f \times \Big(\sigma_{Rf,t}(a_{Rt}; p_t) \times \sigma_{Ds,t}(a_{Dt}; a_{Rt}, p_t)\Big) + (1-f) \times \Big(\sigma_{Df,t}(a_{Dt}; p_t) \times \sigma_{Rs,t}(a_{Rt}; a_{Dt}, p_t)\Big) \quad (15)$$

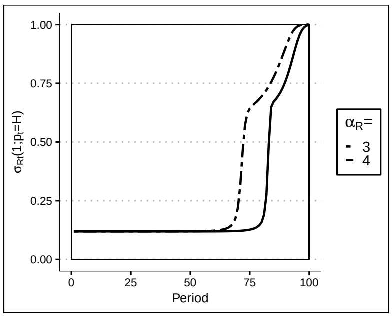
In addition to allowing us to compute CCPs prior to the order of play Proposition 3.2 also gives us sample prediction which Proposition 3.1 fails to do. Reason being, we do not ob-



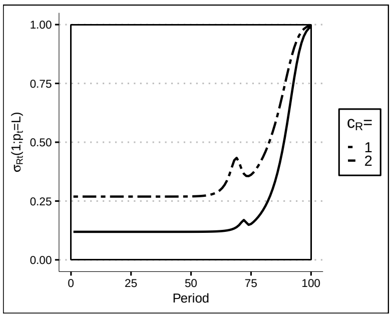

(a) $c_R \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. Increasing the cost introduces a level shift in CCPs. Importantly CCPs decrease in either popularity level.*



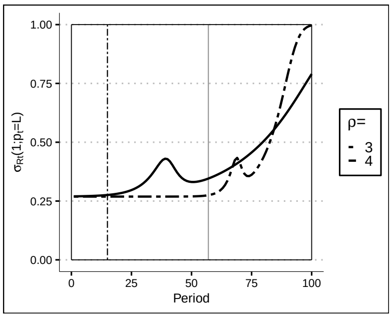

(b) $\rho \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. The vertical lines highlight when a candidate begins to rally aggressively. Higher the persistence in popularity, higher is the eagerness to rally aggressively.*



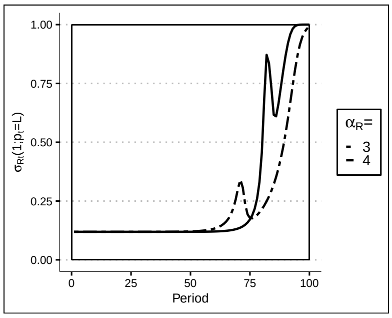

(c) $\alpha_R \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. Changing R's effectiveness affects R's behavior only towards the end of election period. There is delay in when R would choose to become aggressive.*



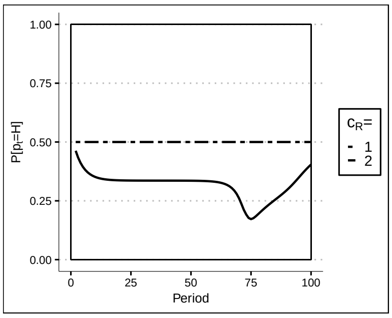

(d) $c_R \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. Increasing the cost introduces a level shift in CCPs. Importantly CCPs decrease in either popularity level.*



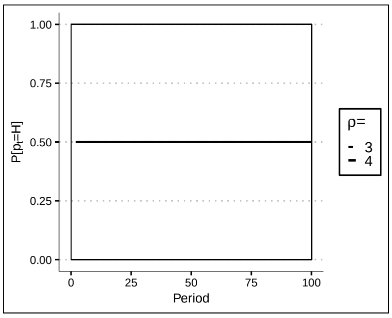

(e) $\rho \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. The vertical lines highlight when a candidate begins to rally aggressively. Higher the persistence in popularity, higher is the eagerness to rally aggressively.*



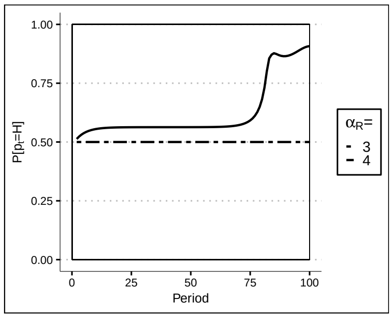

(f) $\alpha_R \uparrow$:
*Here we plot R's equilibrium CCPs prior to order of play given in eq. 16. Changing R's effectiveness affects R's behavior only towards the end of election period. There is delay in when R would choose to become aggressive.*




(g) $c_R \uparrow$:
*Here we plot the unconditional probability with which popularity will realize a H draw along the equilibrium path. Increasing the cost introduces a level shift in CCPs. This reduction then reduces ex-ante probability of being popular.*




(h) $\rho \uparrow$:
*Here we plot the unconditional probability with which popularity will realize a H draw along the equilibrium path. Changing persistence doesn't affect the chances of winning, ceteris paribus.*




(i) $\alpha_R \uparrow$:
*Here we plot the unconditional probability with which popularity will realize a H draw along the equilibrium path. Changing R's effectiveness changes these unconditional probabilities all throughout the election and ensures that chances of R winning is substantially high towards the end.*

Figure 2: *In (a), (b) and (c) we exhibit how R's CCPs respond to higher cost of rallying, persistence in popularity and R' effectiveness respectively. In (g),In (h) and In (i) we exhibit how the unconditional probability of having H draw changes as R's cost of rallying, persistence in popularity and R' effectiveness increases respectively.*

serve which candidate was the first mover, but only what was chosen by the candidates. Moreover, by using this proposition we can compute CCPs for both candidates prior to to order of play. That is the ex-ante probability with which a candidate rallies in state $k$ before realizing whether he is a first mover or the second mover. These probabilities are defined in eq. 16.

$$\begin{aligned}\sigma_{Rt}(k;p_t) &= \sum_{l=0,1,\ldots,K} \sigma_t(k,l;p_t) \\ \sigma_{Dt}(l;p_t) &= \sum_{k=0,1,\ldots,K} \sigma_t(k,l;p_t)\end{aligned} \tag{16}$$

Moreover, to keep results easy to interpret we consider a simplified model where a candidate only decides to rally or not to rally. This simplified model can be obtained by fixing $K = 1$ in our more general model. Proposition 3.1 still applies to this case as only the number of options have been altered by keeping the information and decision making structure intact. We carry-out our comparative static exercises on this simplified model. In this analysis the base parameter values are fixed so that both candidates are symmetric. Their costs of rally are $c_R = c_D = 1$ and their rally effectiveness is given by $\alpha_R = \alpha_D = 3$. The persistence parameter is fixed at $\rho = 3$. The results from these comparative statics are shown in Figures 2 and 3 and a detailed discussion with respect to each parameter is provided below:

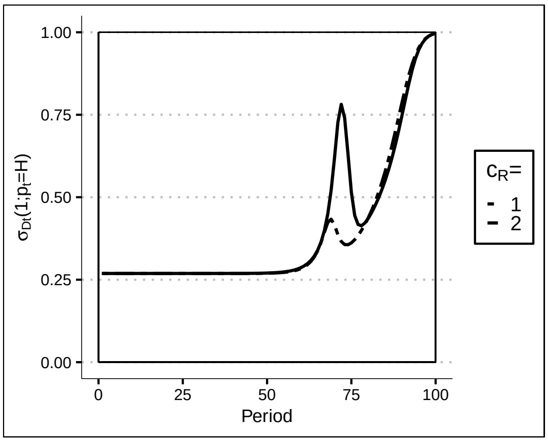
$c_R \uparrow$ Consider the Figures 2a and 2d. Here we have plotted $R$'s equilibrium CCPs as given in eq. 16. These are the CCPs before the order of play is realized. Increasing the cost of rallies for $R$ from $c_R = 1$ to $c_R = 2$ decreases the choice probabilities. There is a vertical shift in the CCPs simply because the candidate is willing to sacrifice higher parliamentary seats in order to not rally and needs more compensation for rallying. Therefore making rallies by the candidate less frequent.

Consider Figure 2g. Here we plot unconditional probability of drawing popularity level $H$. When cost of rallies is higher for $R$, chances of $R$ being popular decreases along the equilibrium path, ceteris paribus. This is driven by lower probability of rallying by $R$ than his opponent. Since, it is costlier for $R$ to rally he would invest less in rallies. If both $R$ and $D$ are equally effective then the one who rallies more will have a higher chance of being popular.

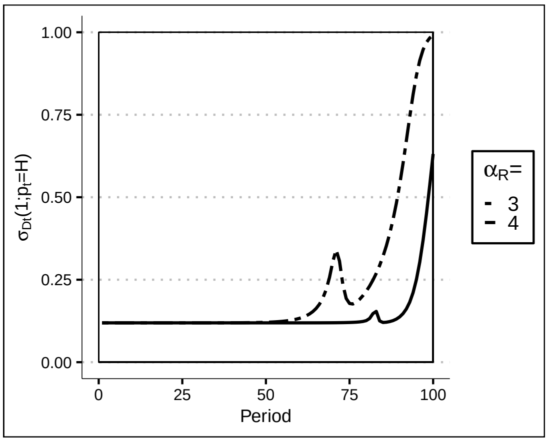
Consider the Figures 3a and 3c we plot $D$'s equilibrium CCPs prior to order of play given in eq. 16. Increasing the cost for $R$ doesn't affect $D$'s initial behavior much. However towards the end he is more likely to rally in state $H$ (not preferred by $D$) than otherwise. There is a marginal decrease in likelihood to rally in state $L$. It is safe to say changing the opponent's cost has marginal effects towards the end of the game.

$\rho \uparrow$ Consider the Figures 2b and 2e. Here we again plot $R$'s equilibrium CCPs given in eq. 16. The vertical lines highlight when a candidate begins to rally more aggressively. That is when the candidate begins to gradually make his rallies more frequent than at the beginning of the game. The comparative static indicates that higher is the persistence in popularity, higher is the eagerness to rally aggressively.

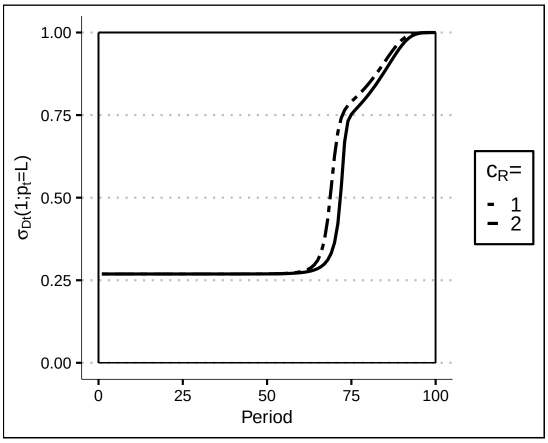
Intuition behind this result highlights the interaction of candidates and their popularity. High persistence in popularity gives candidates higher returns to early rallies than



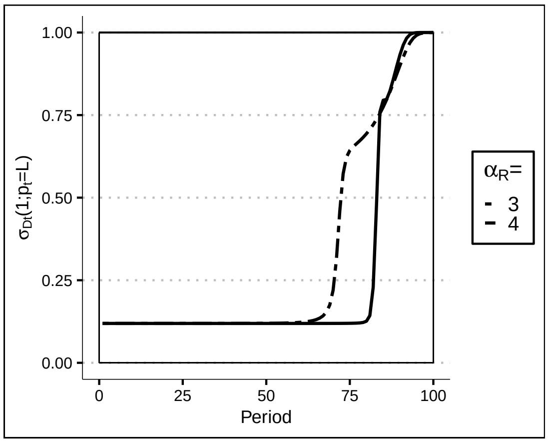

(a) $c_R$ ↑: Here we plot $D$'s equilibrium CCPs prior to order of play given in eq. 16. Increasing the cost for $R$ doesn't affect $D$'s initial behavior much. Changing the opponent's cost has marginal effects towards the end of the game.




(b) $\alpha_R$ ↑: Here we plot $D$'s equilibrium CCPs prior to order of play given in eq. 16. Changing $R$'s effectiveness affects $D$'s towards the end of election period. There is a delay in when $D$ would choose to become aggressive. Moreover, $D$ would not necessarily rally with probability 1 in period $T$.




(c) $c_R$ ↑: Here we plot $D$'s equilibrium CCPs prior to order of play given in eq. 16. Increasing the cost for $R$ doesn't affect $D$'s initial behavior much. Changing the opponent's cost has marginal effects towards the end of the game.



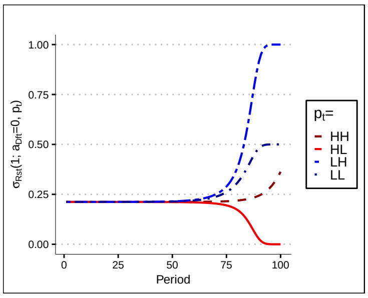

(d) $\alpha_R$ ↑: Here we plot $D$'s equilibrium CCPs prior to order of play given in eq. 16. Changing $R$'s effectiveness affects $D$'s towards the end of election period. There is a delay in when $D$ would choose to become aggressive. Moreover, $D$ would not necessarily rally with probability 1 in period $T$.

Figure 3: *In (a), (b) and (c) we exhibit how R's CCPs respond to higher cost of rallying, persistence in popularity and R' effectiveness respectively. In (g),In (b) and In (c) we exhibit how the unconditional probability of having H draw changes as R's cost of rallying, persistence in popularity and R' effectiveness increases respectively.*

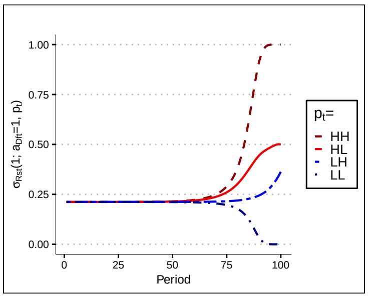
in the case of lower persistence in popularity. Here the persistence parameter captures how volatile is public opinion. In societies with volatile public opinion early rallies will not be beneficial enough as future popularity levels are less predictable. In societies with low volatility of public opinion later rallies might not be beneficial enough as it will require greater effectiveness to change the status quo in one's favor. Therefore, earlier rallies would in turn provide a better chance of ensuring favorable popularity towards the end.

Consider the Figure 2h. Here we plot unconditional probability of drawing $H$. Since the individual specific parameters like cost $c_i$ and $\alpha_i$ are same, probability of drawing $H$ remains unchanged.

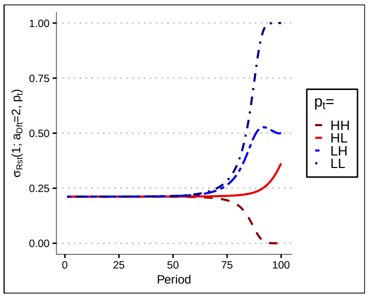
$\alpha_R$ ↑ Consider the Figures 2c and 2f, by changing $R$'s effectiveness from $\alpha_R = 3$ to $\alpha_R = 4$ $R$'s rally behavior changes only towards the end of the game. Initial behavior remains un-



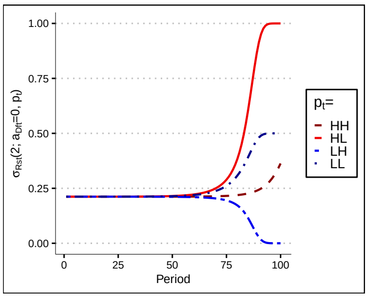

(a) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(1; a_{Dft} = 0, p_t)$, i.e. second mover conditional choice probability for rallying in state 1 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover did not rally.*



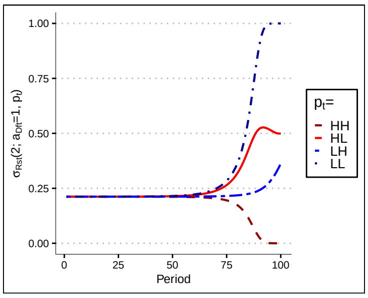

(b) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(1; a_{Dft} = 1, p_t)$, i.e. second mover conditional choice probability for rallying in state 1 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover chose state 1 for rally.*



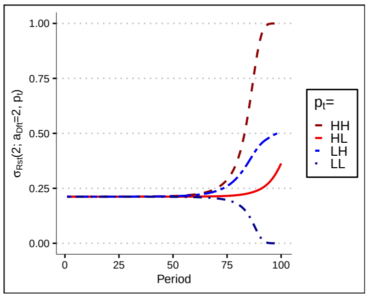

(c) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(1; a_{Dft} = 2, p_t)$, i.e. second mover conditional choice probability for rallying in state 1 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover state 2 for rally.*




(d) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(2; a_{Dft} = 0, p_t)$, i.e. second mover conditional choice probability for rallying in state 2 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover did not rally.*




(e) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(2; a_{Dft} = 1, p_t)$, i.e. second mover conditional choice probability for rallying in state 2 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover chose state 1 for rally.*




(f) *On the $x-$axis we have periods while on the $y-$axis we have $\sigma_{Rst}(2; a_{Dft} = 2, p_t)$, i.e. second mover conditional choice probability for rallying in state 2 for each popularity $p_t \in \{LL, LH, HL, HH\}$ when first mover state 2 for rally.*

Figure 4: *Here, we plot second mover conditional choice probabilities for each choice made by first mover and at each popularity level on the $y-$axis. On the $x-$axis we are plotting periods. Figures 4a and 4d correspond to the case when first mover did not rally. Figures 4b and 4e correspond to the case when first mover chose state 1 for rallies. Figures 4a and 4d correspond to the case when first mover chose state 2 for rally. These plots correspond the case of model 1, i.e. when states are identical.*

altered confirming that it is controlled solely by $c_R$. In Figure 2i we plot unconditional probability of drawing $H$. By making $R$ more effective his chances staying popular improves and towards the end of the game probability of $R$ winning increases even further. In Figures 3b and 3d, we show $D$'s behavior when $R$ is more effective. Here like $R$, $D$ also delays rallying aggressively. In case he is not popular he would not rally with probability one in the last period $T$.

## 3.5 Multiple States: Equilibrium Properties

In this section we will show some important features the model exhibits in equilibrium with multiple states. For this exercise we will consider two configurations of the model, where each configuration will have two states. In the first configuration the states will possess the same number of parliamentary seats and in the second configuration one state will have higher number of parliamentary seats. We will ensure that the sum of parliamentary seats are 150. Moreover, we want to analyze how popularity across states can influence decisions made in the model and for this purpose it is important to ensure that electoral college across states are equal. Hence our first configuration has equal parliamentary seats. Second, we also wish to see how higher number of parliamentary seats in one state would affect choices.

Therefore, in our second configuration one state will have a higher number of parliamentary seats.
The first configuration is referred as model 1 and it will have two states, state 1 and state 2. State 1 and 2 will have equal parliamentary seats, i.e. $E_1 = E_2 = 75$. The second configuration is referred as model 2 but state 1 will have higher parliamentary seats than state 2, i.e. $E_1 = 100$ and $E_2 = 50$. In both the models candidates will have equal cost of rallying $c_R = c_D = 1$[13], and equal effectiveness magnitudes, i.e. $\alpha_R = 3$ and $\alpha_D = -3$. The persistence in popularity is set at $\rho = 4$ to ensure candidates have enough eagerness to rally early[14]. In both configurations, model 1 and model 2 the total number of periods where candidates make decisions is going to be set to 100. That is $T = 100$ in both configurations.

### 3.5.1 Discussion on Equilibrium Strategies

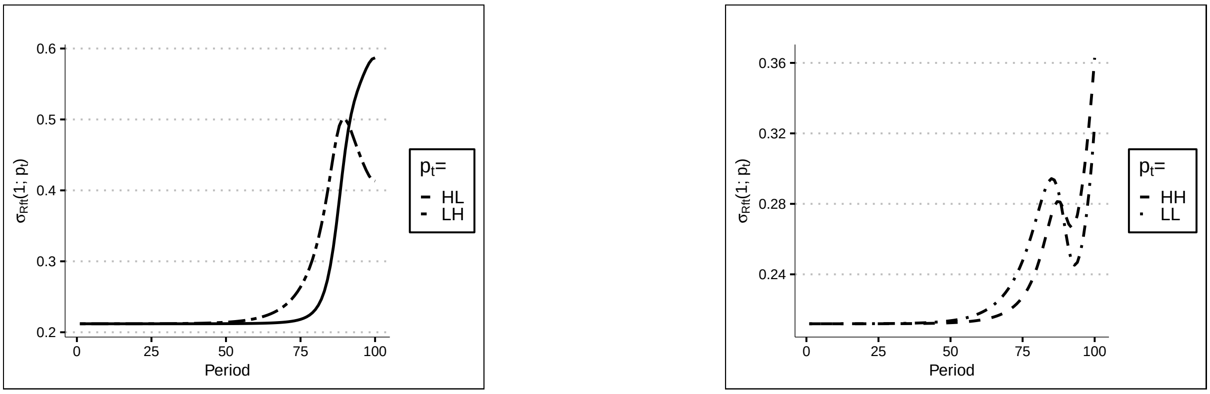
First consider, second mover's equilibrium responses for model 1. Second mover's equilibrium responses depend on the current popularity across states and first mover's chosen action. Here, first mover is going to be $D$ and the second mover is going to be $R$.

$a_{Dft} = 0$ First consider the case where $D$ doesn't rally. Here, $R$'s responses will be different based on popularity across states (refer to the Figures 4a and 4d). The rationale for different responses at each popularity level are provided below:

- *HH* If popularity is *HH*, i.e. $R$ is popular in both states, then $R$ would exhibit strategic complementarity in both states. Since $D$ did not rally in a state, $R$ would also prefer to compliment that by not rallying in that state. Due to this, $R$ would prefer to not rally in either states. As a result, probability of rallying in either state will be low and only increase when elections are very close.
- *LL* Unlike the previous case, here $R$ would exhibit strategic substitution. If popularity is *LL*, then $R$ would exhibit strategic substitution in both states and hence would rally in state 1 and 2 with equal probability. Overall probability of rally is high, however rallying in one particular state will reach $\frac{1}{2}$.
- *HL* If popularity is *HL*, then $R$ exhibits strategic complementarity in state 1 and strategic substitution in state 2. This compels $R$ to not rally in state 1 and rally in state 2. Therefore, $R$ would rally intensely in state 2 and not rally state 1.
- *LH* This case is symmetric to the previous case. Here the opposite would happen, i.e.$R$ will exhibit strategic substitution in state 2 and complementarity in state 1. As a result $R$ would rally intensely in state 1.

[13] In our calibrations (omitted here) showed that setting the cost parameters close to 1 were good approximations for rationalizing initial rally decisions.

[14] With a value for $\rho = 3$, the rally decisions were not sufficiently early enough. As a result graphical analysis is harder to carry out as conditional choice probabilities come too close to each other and are harder to distinguish visually.




(a) On the $x-axis$ we have periods while on the $y-axis$ we have $\sigma_{Rft}(1;p_t)$, i.e. second mover conditional choice probability for rallying in state 1 for each popularity $p_t \in \{LH, HL\}$. The conditional choice probability of rallying in state 2 is symmetric. That is $\sigma_{Rft}(2;p_t = HL) = \sigma_{Rft}(1;p_t = LH)$ and $\sigma_{Rft}(2;p_t = LH) = \sigma_{Rft}(1;p_t = HL)$

(b) On the $x-axis$ we have periods while on the $y-axis$ we have $\sigma_{Rft}(1;p_t)$, i.e. second mover conditional choice probability for rallying in state 1 for each popularity $p_t \in \{HH, LL\}$. The conditional choice probability of rallying in state 2 is identical. That is $\sigma_{Rft}(2;p_t = HH) = \sigma_{Rft}(1;p_t = HH)$ and $\sigma_{Rft}(2;p_t = LL) = \sigma_{Rft}(1;p_t = LL)$

Figure 5: Here, we plot periods on the $x-axis$ and $\sigma_{Rft}(1;p_t)$, i.e. first mover conditional choice probability of rallying in state 1 on the $y-axis$. These plots correspond to model 1 that is the case of identical states. In Figure 5a we plot probability of rallying in state 1 when popularity is either *HL* or *LH*. The probability of rallying in state 2 under *HL* and *LH* is symmetric to Figure 5a. In Figure 5b we plot probability of rallying in state 1 when popularity is either *HH* or *LL*. The probability of rallying in state 2 under *HH* and *LL* is symmetric to Figure 5b.

$a_{Dft} = 1$ Suppose $D$ as a first mover rallies in state 1. Then $R$'s responses will be different contingent on popularity across states (refer to the Figures 4b and 4e). The rationale behind the differing response is given below:

*HH* If popularity is *HH*, then $R$ exhibits strategic complementarity in both states. This would compel $R$ to rally in state 1, as $D$ rallied there, and not to rally in state 2, as $D$ did not rally there. As a result, $R$ rallies intensely in state 1.

*LL* If popularity is *LL*, the opposite happens as $R$ would exhibit strategic substitution in both states. Since $D$ rallied in state 1 and not in state 2, $R$ would prefer to rally in state 2 over state 1. Therefore, rally intensely in state 2 as opposed to rally in state 1.

*HL* If popularity is *HL*, then there is strategic substitution in state 2, therefore $R$ is willing to rally in state 2. There is strategic complementarity in state 1 and therefore $R$ wishes to rally in state 1. Probability of rally is high, however, probability of rallying in a particular state is suppressed.

*LH* If popularity is *LH*, then there is strategic substitution in state 1 that discourages rallying in state 1 and there is strategic complementarity in state 2 that also discourages rallying in state 2. Probability of any rally is low and only increases because value from not rallying decreases with time.

$a_{Dft} = 2$ The case when $D$ rallies in state 2 is symmetric to the case when opponent rallies in state 1.

First mover's behavior depends on his value from rallying as a second mover and the probability with which he expects his opponent to respond. Suppose $R$ is the first mover and the current popularity is $LH$, please refer to Figure 5a, then he wishes to not rally in state 1 if $D$ rallies there and rally in state 2 if $D$ rallies there. While $D$ wishes to rally in state 1 if $R$ rallies there and not rally in state 2 if $R$ rallies there. By rallying in state 1 he encourages $D$ to diversify his rallies. As, $D$ wishes to rally in state 1 because $R$ rallied there but also rally in 2 as $R$ did not rally there[15]. As a result $D$'s response in state 1 and 2 is going to be suppressed. If $R$ rallies in state 2, then $D$ would like to avoid rallying in 2 and 1 both[16]. As a result $D$'s rallies are going to be low in this case and only increases later in time. As it turns out, initially $R$ has more to gain from letting $D$ diversify so that $R$ can attempt to change the status quo. However, as we come close to elections $R$ strongly prefers to maintain the status quo of staying popular in state 1 and unpopular in state 2. A similar explanation can be made for the case when popularity is $HL$.

When popularity is $HH$, $R$'s opponent exhibits strategic substitution in both the states. If $R$ would rally in a state, his opponent would rally with high probability in the other state. Since, $R$ exhibits strategic complementarity in both the states, he would also prefer to rally in the other state as well. Therefore, his conditional choice probability from rallying in a particular state is lower. If popularity is $LL$, his opponent would rally in the same state as $R$ with a high probability. Since, $R$'s gains exhibit strategic substitution, he would attempt to diversify his rally so that his opponent would not follow. This suppresses his rallying choice in a particular state.

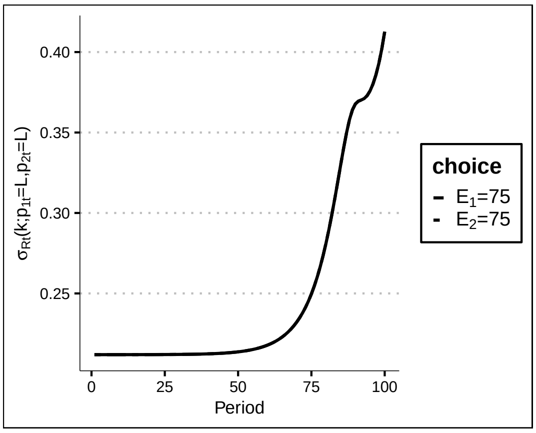
Now, we compare conditional choice probabilities of model 1, where states are identical, to that of model 2, where one state has higher number of parliamentary seats. For brevity we will compare conditional choice probabilities prior to the order of play as given by eq. 16. Here, in addition to the subtle mechanisms which we discussed before, number of parliamentary seats play a big role as well. We compare the decisions in the models for each popularity level. We organize them below:

- *LL* Consider the case when popularity is $LL$ (please refer to Figures 6a and 6b). Here, in model 1 (when states are identical) $R$ rallies in both states with equal probability. This is justified as both states are identical. However, in the case of different states a $R$ initially rallies more in the state with higher parliamentary seats and later in the state with lower parliamentary seats. Since state 1 has higher parliamentary seats it has a higher value to $R$ and therefore challenging the status quo in state 1 is justified. However, later $D$ begins to rally intensely in state 1 with higher parliamentary seats. This makes $R$'s rallies in state 1 less beneficial and rallies in 2 more beneficial. As a result $R$ prefers to challenge status quo in state 2 at this stage.

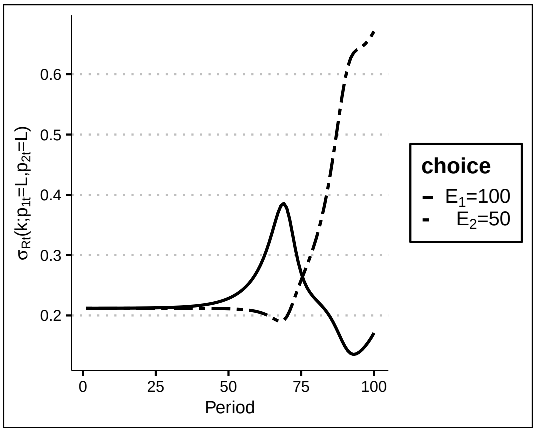
- *LH* Consider the case when popularity is $LH$, i.e. $R$ is popular in state 2 and unpopular in the state 1. For this please refer to Figures 6c and 6d. Here in model 1, that is when

[15] In the discussion for second mover this is the case where $D$ rallied in 1 and popularity is $HL$

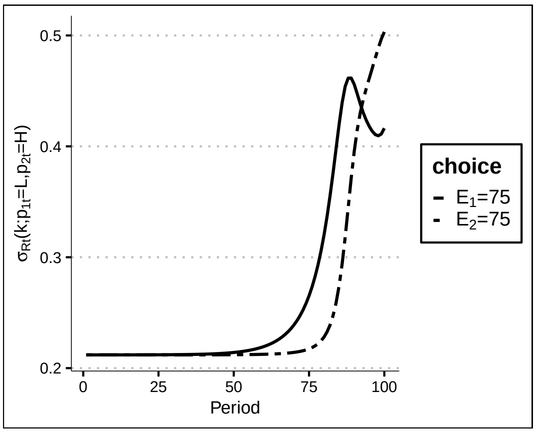
[16] In the discussion for second mover this is the case where $D$ rallied in 1 and popularity is $LH$



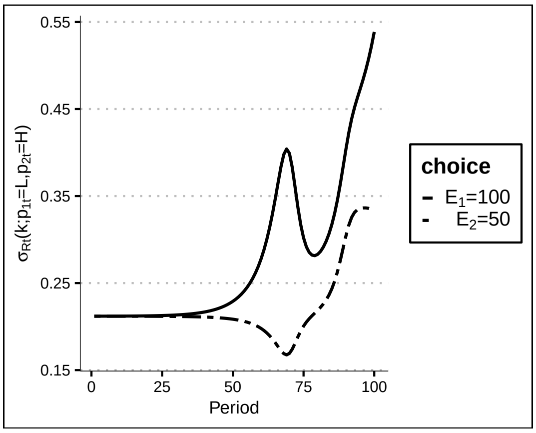

(a) *In this figure we only consider $p_t = LL$ when states are identical, i.e. model 1. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.*




(b) In this figure we only consider $p_t = LL$ when state 1 has higher number of parliamentary seats, i.e. model 2. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.




(c) In this figure we only consider $p_t = LH$ when states are identical, i.e. model 1. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.

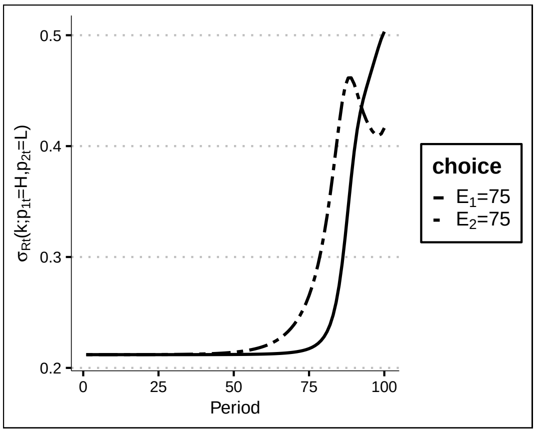



(d) In this figure we only consider $p_t = LH$ when state 1 has higher number of parliamentary seats, i.e. model 2. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.

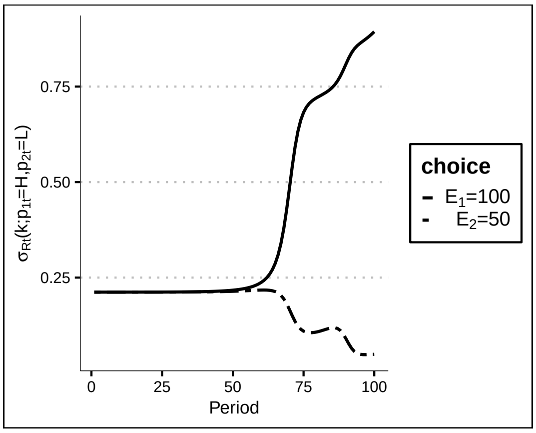
Figure 6: Here we showing $R$'s Conditional choice probabilities prior to order of play ($\sigma_{Rt}(k, p_t)$ as defined in eq. 16) on $y - axis$. We have periods on the $x - axis$. Here Figures 6a and 6c consider $\sigma_{Rt}(k, p_t)$ when states are identical at different popularity levels. Figures 6b and 6d consider $\sigma_{Rt}(k, p_t)$ when states are different at different popularity levels.

states are identical, $R$ rallies more in the state where he is unpopular initially and then later move towards maintaining the status quo by rallying more in the second state. However, in model 2 that is when state 1 has higher number of parliamentary seats, the potential benefit from becoming popular in state 1 overpowers this. Therefore, if a candidate is unpopular in state 1 he still rallies more in the state where he is unpopular and slightly increase his probability of rallying in the other state. This is in contrast to the case of identical states. Therefore, a high number of parliamentary seats in one state can dominate the channels through which strategic substitution and complementarity govern the decisions.

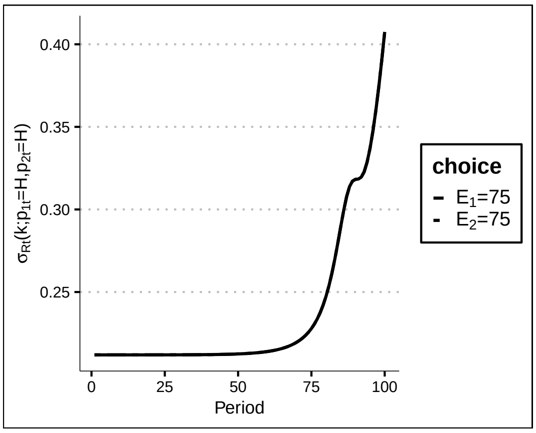
*HL* When popularity is $HL$, in model 2, $R$ is popular in the state with higher parliamentary seats and unpopular in the state with lower parliamentary seats (refer to Figures 7a and 7b). In model 1, that is identical states, $R$ follows a strategy that is symmetric to the case of $LH$. However, in model 2 if the states are not identical then $R$ prefers to maintain the status quo in state 1. This is a result of two things, first there is strategic



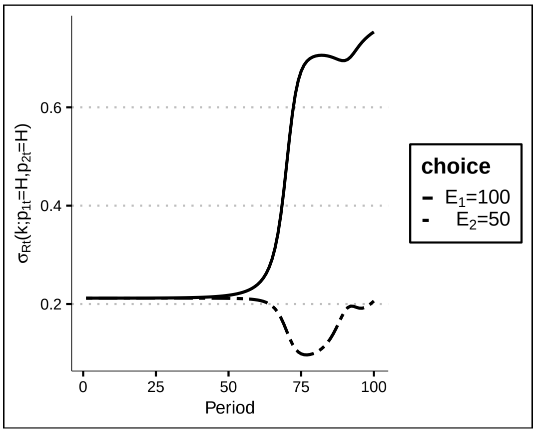

(a) In this figure we only consider $p_t = HL$ when states are identical, i.e. model 1. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.




(b) In this figure we only consider $p_t = HL$ when state 1 has higher number of parliamentary seats, i.e. model 2. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.




(c) In this figure we only consider $p_t = HH$ when states are identical, i.e. model 1. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$(defined in eq. 16) for each $k \in \{1, 2\}$.

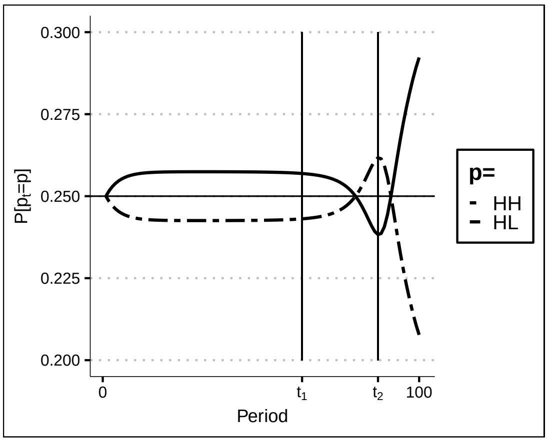



(d) In this figure we only consider $p_t = HH$ when state 1 has higher number of parliamentary seats, i.e. model 2. Here we plot periods on $x - axis$ and on $y - axis$ we are plotting $\sigma_{Rt}(k, p_t)$ (defined in eq. 16) for each $k \in \{1, 2\}$.

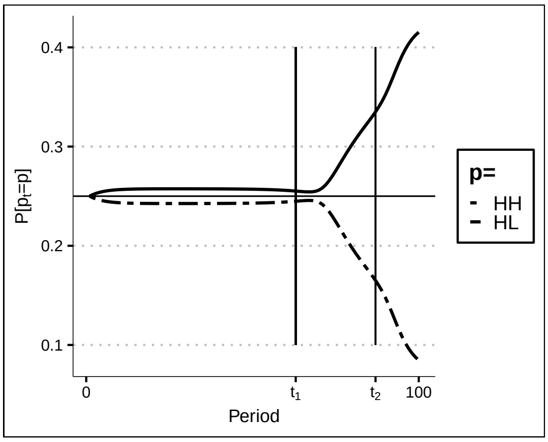
Figure 7: Here we showing $R$'s Conditional choice probabilities prior to order of play ($\sigma_{Rt}(k, p_t)$ as defined in eq. 16) on $y - axis$. We have periods on the $x - axis$. Here Figures 7a and 7c consider $\sigma_{Rt}(k, p_t)$ when states are identical at different popularity levels. Figures 7b and 7dconsider $\sigma_{Rt}(k, p_t)$ when states are different at different popularity levels.

complementarity in state 1. Here, $D$ is unpopular in state 1 and popular in state 2, therefore similar to previous case $D$ wishes to rally in state 1. This motivates $R$ to rally in state 2. Second reason is the high number of parliamentary seats dominate strategic complementarity which $R$ exhibits in state 2 that would make $R$ to challenge status quo in state 2 (as in the case of identical states).

*HH* Similar, tendency can be seen when popularity is $HH$ (please refer to Figures 7c and 7d). In case of model 1, that is the identical states model, both states are given equal importance. In model 2, candidate strictly prefers to maintain the status quo in state 1 and is willing to sacrifice parliamentary seats in state 2. Moreover, unlike the case when popularity is $HL$, there is slight increase in probability of rallying in state 2 towards the end. This is because $D$ begins to rally in state 2 intensely. As $R$ also exhibits strategic complementarity in state 2 he increases his rallies in state 2. However, due to high number of parliamentary seats in state 1, gain from maintaining the status quo in state 1 dominates the gains from maintaining the status quo in state 2. This increases




(a) This Figure plots $P[p_t = p]$ as defined in eq. 18 for $p \in \{HH, HL\}$ on the equilibrium path for model1. Both states have same number of parliamentary seats. Probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$. Periods $1 - t_1$ shows the region where $HL$ and $LH$ are more probable than $HH$. In periods $t_1 - t_2$ $HH$ and $LL$ become more probable than $HL$ or $LH$. From period $t_2 - 100$ $HL$ and $LH$ becomes more probable again.




(b) This Figure plots $P[p_t = p]$ as defined in eq. 18 for $p \in \{HH, HL\}$ on the equilibrium path for model 2. State 1 has higher number of parliamentary seats than state 2. Probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$. Unlike model 1 (Figure 8a), $LH$ and $HL$ is always more probable than $HH$ and $LL$.

Figure 8: Figure 8a shows unconditional probability of popularity vector $p \in \{HH, HL\}$ for model 1, i.e. when states that have identical parliamentary seats. Figure 8b shows unconditional probability of popularity vector $p \in \{HH, HL\}$ when state 1 has higher parliamentary seats and candidates are equally effective.

the probability of rallying in state 2 by $R$ only marginally.

### 3.5.2 Discussion on Equilibrium Popularity

Now, we shall discuss unconditional probability with which popularity takes a value $p$ along the equilibrium path. For this exercise please refer to Figures 8a and 8b. Figure 8a corresponds to model 1, where states are identical, and Figure 8b corresponds to model 2, where state 1 has higher parliamentary seats. In both cases initially between periods 0 and $t_1$ we see that popularity $HL$ is more probable than $HH$ (please note $P[p_t = LH] = P[p_t = HL]$ and $P[p_t = HH] = P[p_t = LL]$). However, between periods $t_1$ and $t_2$ in model 1 $HH$ becomes more probable than $HL$. Between periods $t_2$ and 100 $HL$ again becomes more probable. The same doesn't occur in model 2. The reasons for this reversal in case of model 1 and no such reversal in model 2 can be explained by strategies being used in both cases.

To understand where these strategies come into play first consider probability of entering popularity $p'$ conditioned on the event that current popularity is $p$ as $q^{p}_{p',t}$. On the equilibrium path this probability is given by the following expression:

$$q^{p}_{p',t} = \sum_{k \in K} \sum_{l \in K} P\Big(p_{t+1} = p' \Big| a_{Rt} = k, a_{Dt} = l, p_t = p\Big) \times \sigma_t\Big(a_{Rt} = k, a_{Dt} = l, p_t = p\Big) \tag{17}$$

Here, $\sigma_t(a_{Rt} = k, a_{Dt} = l, p_t)$ was defined in Proposition 3.2. This is the probability of $R$ rallying in state $k$ and $D$ rallying in state $l$ when popularity $p_t$ in period $t$ takes the value $p$. Moreover, $P\Big(p_{t+1} = p' \Big| a_{Rt} = k, a_{Dt} = l, p_t = p\Big)$ was defined in eq. 3. Clearly, by definition $q^{p}_{p',t}$ depends on equilibrium choice probabilities at each period $t$. These probabilities govern the transitions of popularity from $p$ to $p'$. Now, we define $P[p_t = p']$ as the unconditional probability of popularity $p'$ at given period $t$. It is possible to compute this probability recursively as followed:

$$P[p_t = p'] = \sum_{p \in \{H,L\}^K} q^{p}_{p',t-1} \times P[p_{t-1} = p] \tag{18}$$

Here, $P[p_{t-1} = p]$ is the unconditional probability of popularity level $p$ in period $t-1$. We plot the unconditional probabilities for popularity level $p \in \{HH, HL\}$ in the Figure 8a for the case of identical states, that is model 1 and for model 2 in Figure 8b.

model 1 When states are identical then probability of one candidate winning in one state and loosing the other ($HL$ or $LH$) remains high as opposed to one loosing in both ($LL$) or winning in both ($HH$). However, this does not remain true over all periods. Please consider the following sets of periods separately:

$1 - t_1$ In these periods, candidates value all states equally irrespective of their popularity. At any given period and popularity, probability of one candidate succeeding in being popular in one state is higher than him becoming popular in both states. As a result, probability of one candidate being popular in one state and unpopular in the other remains higher.

$t_1 - t_2$ In these periods, candidates attempt to change the status quo in the state they are unpopular (refer to Figures 6c and 7a). However, probability of one candidate succeeding is higher than both succeeding at the same time. As a result probabilities $q^{HL}_{HH,t}$, and $q^{HL}_{LL,t}$ are higher than $q^{HL}_{LH,t}$. Moreover, the probability with which status quo is maintained ($q^{HL}_{HL,t}$) is also lower. These factors, which arise by increased tendency to change the status quo, lead to $HL$ becoming less probable and $HH$ more probable.

$t_2 - 100$ Unlike the previous set of periods, here candidates prefer to maintain the status quo when popularity is $HL$. As a result now $q^{HL}_{HH,t}$ and $q^{HL}_{LL,t}$ is low. While the probability with which status quo is maintained ($q^{HL}_{HL,t}$) is higher. These factors arise because candidates prioritize maintaining the status quo or in other words securing their win in states where they are already popular. Hence $HL$ again becomes more probable and $HH$ less probable.

A similar explanation can given for $LH$ and $LL$. Since with symmetric candidates these probabilities are identical we omit this case here.

model 2 Now consider the case when parliamentary seats are different, that is model 2, (consider the Figure 8b). The dynamics are different because candidates use different strategies in model 2 when compared to model 1. If popularity is $HL$, then only $D$ is interested in changing the status quo (this is identical to the case for $R$ under popularity $LH$ in Figure 6d) while $R$ is interested in maintaining the status quo. This achieves two things. First, since $R$ is not attempting to change the status quo in state 2, $q^{HL}_{HH,t}$ is low. Second, $R$ is acting against $D$ to maintain the status quo in state 1. Here $q^{HL}_{LL,t}$ is not sufficiently high. Therefore, $HH$ and $LL$ are less probable while $HL$ is more probable. In case popularity was $LH$, $R$ and $D$ switch their roles and $LH$ stays more probable. Therefore, $HH$ or $LL$ do not become more probable when compared to $LH$ or $HL$.

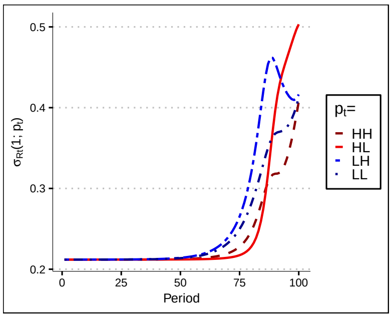
The channels of strategic substitution and strategic complementarity are important for governing the behavior of a candidate in choosing states to rally. In model 1 these become less important only towards the end of the game when elections are very close but still justify the magnitudes across different popularity values. In case of model 2, these channels are weaker and behavior is governed mostly by the distribution of parliamentary seats. Candidates in this case value the states with high parliamentary seats much more than strategic advantages they might have and therefore show tendencies that violate the rationale of strategic complementarity or substitution. However, these channels are not completely diminished and do govern the behavior marginally and can justify the magnitudes.

In this model, behavior across states with similar number of parliamentary seats will be governed by the channels of strategic substitution. However, when parliamentary seats are skewed towards specific states, these specific states are given higher importance than the ones that have lower number of parliamentary seats. Therefore, when one has electoral college vote distribution skewed over specific states then strategic elements will play a relatively marginal role.

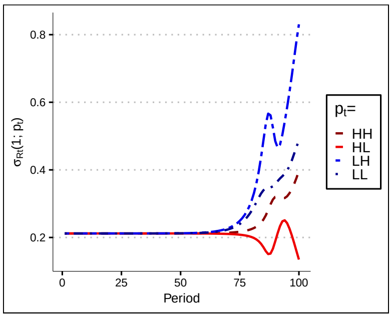
## 3.6 Multiple States: Comparative Statics

In order to understand what happens when we change parameters $c_i$, $\alpha_i$ and $\rho$, we will consider model 1 as the base model for all three parameters. We wish to focus on model 1 in order to avoid the effects that arise when one state has higher number of parliamentary seats. We organize the remainder of this section in three parts. First will discuss comparative static with respect to candidate effectiveness parameter ($\alpha_R$), second will discuss comparative static with respect to cost parameter ($c_R$) and third will discuss comparative static with respect persistence parameter ($\rho$).

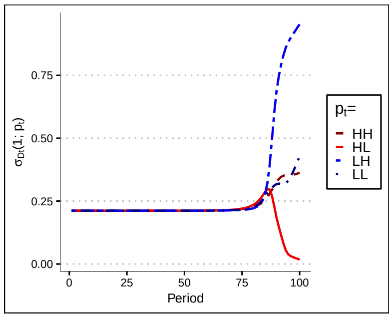
$\alpha_R \uparrow$ Here we discuss what happens when one candidate is more effective than the opponent. We consider the model where states are identical. For this exercise we will set $\alpha_R = 4$ and $\alpha_D = -3$ for model 1. Here, $R$ is more effective than $D$. The chances of winning in each state are shown in Figure 9d for the case candidates are equally effective and the Figure 9e for the case when $R$ is more effective. When they are equally effective, the unconditional probability of $HL$ and $LH$ remains higher than $HH$ and $LL$ for majority of the game (reasons discussed in the previous section). When $R$ is more effective, from the beginning $HH$ is the most probable and then later $LH$ or $HL$



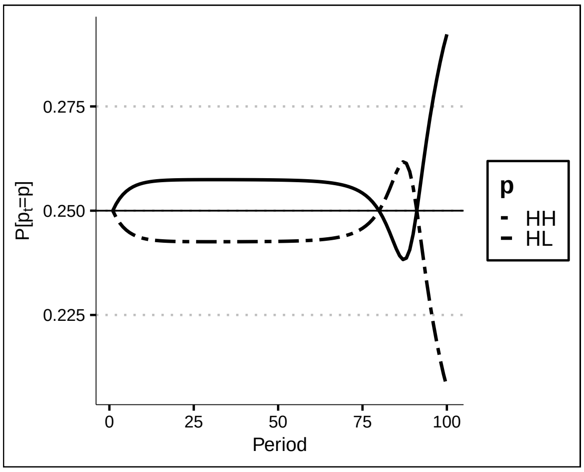

(a) The Figure plots $\sigma_{Rt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats and candidates are equally effective. Since states are identical $\sigma_{Rt}(2, p_t)$ is symmetric: $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p')$ for $p = HL$(or $p = LH$) and $p' = LH$(or $p' = HL$).



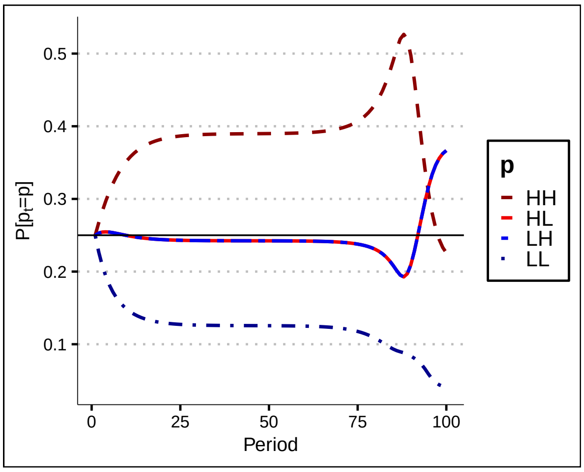

(b) The Figure plots $\sigma_{Rt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats, however $R$ ($alpha_R = 4$) is more effective than $D$ ($alpha_D = -3$). Since states are identical $\sigma_{Rt}(2, p_t)$ is symmetric: $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p')$ for $p = HL$ (or $p = LH$) and $p' = LH$ (or $p' = HL$).




(c) The Figure plots $\sigma_{Dt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats, however $R$ ($alpha_R = 4$) is more effective than $D$ ($alpha_D = -3$). Since states are identical $\sigma_{Dt}(2, p_t)$ is symmetric: $\sigma_{Dt}(2, p) = \sigma_{Dt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Dt}(2, p) = \sigma_{Dt}(1, p')$ for $p = HL$ (or $p = LH$) and $p' = LH$ (or $p' = HL$).




(d) Here we $P[p_t = p]$ for $p \in \{HH, HL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats and candidates are equally effective. Moreover, probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$.

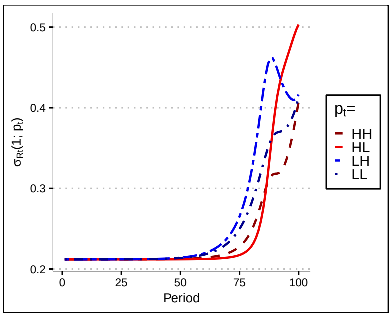



(e) Here we $P[p_t = p]$ for $p \in \{HH, HL, LH, LL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats but $R$ has more effective rallies than $D$.

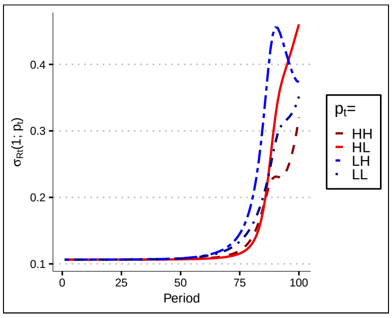
Figure 9: Comparative static for $\alpha_R \uparrow$. Each Figure is meant for model where states have equal parliamentary seats. Figure 9a shows conditional choice probability for $R$ when both candidates are equally effective. Figure 9d shows unconditional probability of popularity vector $p \in \{HH, HL\}$ when states are identical and both candidates are equally effective. Figures 9b and 9c shows conditional choice probabilities for $R$ and $D$ respectively when $R$ is more effective than $D$. Figure 9e shows the corresponding unconditional probability of popularity vector $p \in \{HH, HL\}$.

are more probable than $HH$. Moreover, $LL$ remains the least probable outcome of all of them. The reason behind $HH$ being the most probable is a direct effect of $R$'s effectiveness. Initially $R$ and $D$ both rally with equal probability in both states, however, since $R$ is more effective his popularity is most likely to remain high in both states. The sudden rise in probability of $HH$ and then drop in this probability can be explained by considering the rallying strategies undertaken by candidates.

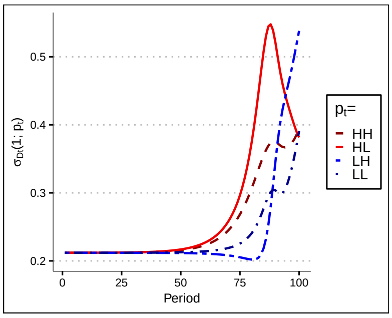
Consider the Figures 9c and 9c. Here $R$ at popularity $LH$ $R$ begins to challenge the status quo in state 1. At this time $D$ is still not responsive and maintains the initial level of probability of rallying. Due to this attempt by $R$ $q^{LH}_{HH,t}$ rises. Therefore, $HH$ becomes more probable. Then $D$ begins to maintain the status quo in state 1, therefore, challenging $R$'s rallies. This leads to a decrease in $q^{LH}_{HH,t}$ and $HH$ becomes less probable while $LH$ becomes more probable. A symmetric explanation can given for why prob-



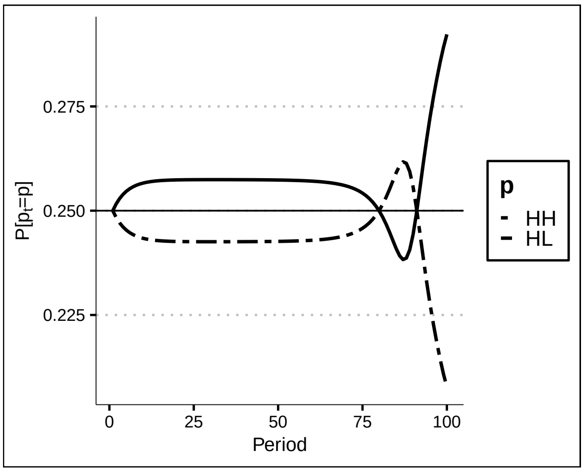

(a) The Figure plots $\sigma_{Rt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats and candidates have equal costs. Since states are identical $\sigma_{Rt}(2, p_t)$ is symmetric: $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p')$ for $p = HL$(or $p = LH$) and $p' = LH$(or $p' = HL$).



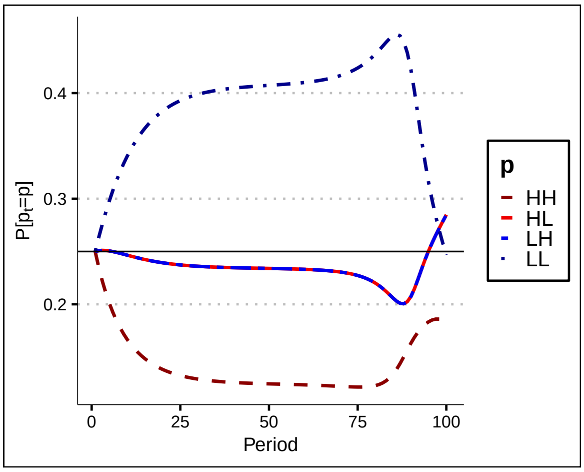

(b) The Figure plots $\sigma_{Rt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats, however $R$ ($c_R = 2$) has higher cost of rallying than $D$ ($c_D = 1$). Since states are identical $\sigma_{Rt}(2, p_t)$ is symmetric: $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Rt}(2, p) = \sigma_{Rt}(1, p')$ for $p = HL$ (or $p = LH$) and $p' = LH$ (or $p' = HL$).




(c) The Figure plots $\sigma_{Dt}(1; p_t)$ (defined in eq. 16). The Figure corresponds to model 1 where both states have same number of parliamentary seats, however $R$ ($c_R = 2$) has higher cost of rallying than $D$ ($c_D = 1$). Since states are identical $\sigma_{Dt}(2, p_t)$ is symmetric: $\sigma_{Dt}(2, p) = \sigma_{Dt}(1, p)$ for $p = HH$ or $p = LL$ and $\sigma_{Dt}(2, p) = \sigma_{Dt}(1, p')$ for $p = HL$ (or $p = LH$) and $p' = LH$ (or $p' = HL$).




(d) Here we $P[p_t = p]$ for $p \in \{HH, HL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats and candidates are equally effective. Moreover, probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$.



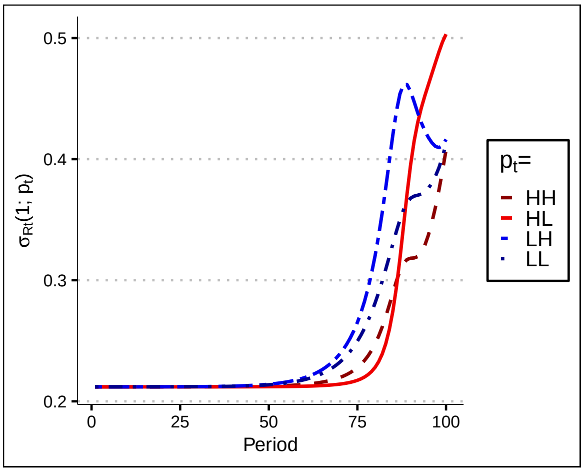

(e) Here we $P[p_t = p]$ for $p \in \{HH, HL, LH, LL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats. However, $R$ has a higher cost of rallying than $D$.

Figure 10: Comparative static for $c_R \uparrow$. Each Figure is meant for model where states have equal parliamentary seats. Figure 10a shows conditional choice probability for $R$ when both candidates face equal cost of rallies. For this case, Figure 10d shows unconditional probability of popularity vector $p \in \{HH, HL\}$. Figures 10b and 10c shows conditional choice probabilities for $R$ and $D$ respectively when $R$ faces higher cost of rallies. For this case, Figure 10e shows unconditional probability of popularity vector $p \in \{HH, HL\}$.

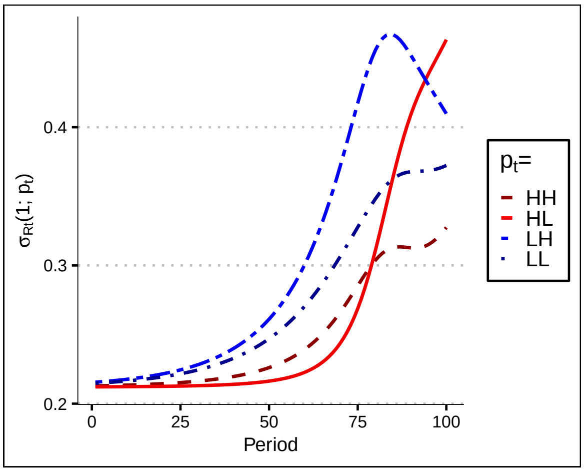
ability $HL$ also rises. Moreover, $R$ is more aggressive at popularity $LL$ than $HH$, which explains why $LL$ keeps decreasing all through out.

$c_R \uparrow$ For this exercise, we consider model 1 and compare it with the case where $R$'s cost of rallying is higher than $D$. That is, $c_R = 2$ and $c_D = 1$ and everything else is identical to model 1. The Figures 10b and 10c show the conditional choice probabilities when $R$ has a higher cost of Rallies than $D$. Since $R$, has a higher cost than $D$, his initial probability of rallying is lower than the case when his cost of rallying was lower. However, since $D$'s cost of rallies is same as before, his initial probability remains the same.

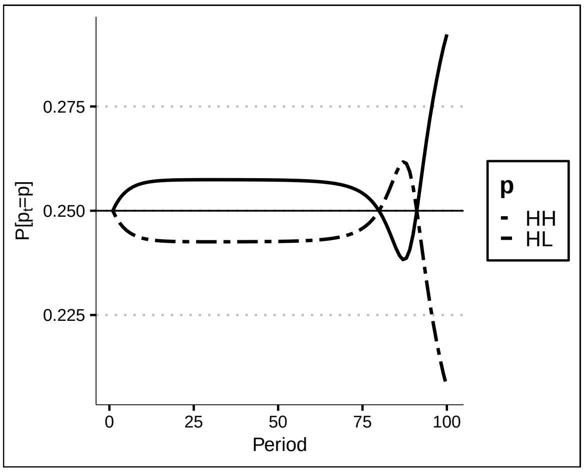
Now, consider the Figures 10d and 10e. When cost of rallying for $R$ is higher than $D$ his initial probability of rallying is also lower than $D$. Due to this lower probability of rallying $R$ has a lower chance of becoming popular in either states than $D$. This is why we see $LL$ to be the most probable outcome. However, later $LH$ or $HL$ becomes



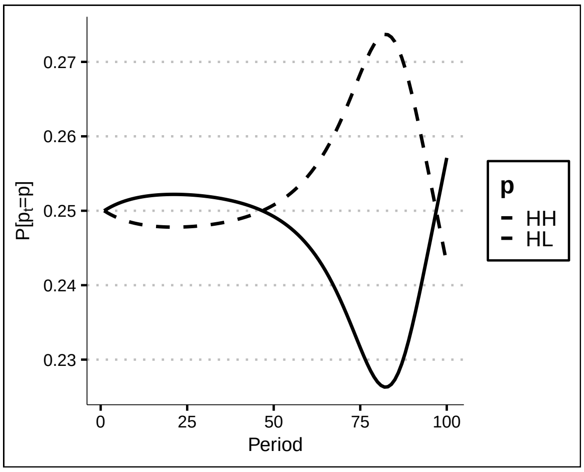

(a) The figure shows a plot of $\sigma_{Rt}(1;p_t)$ for $p_t \in \{HH, HL, LH, LL\}$ as defined in eq. 16 when both states are identical and $\rho = 4$. Probability of rallying in state 2 is symmetric to this figure. Moreover, conditional choice probability for $D$ is symmetric as well.




(b) The figure shows a plot of $\sigma_{Rt}(1;p_t)$ for $p_t \in \{HH, HL, LH, LL\}$ as defined in eq. 16 when both states are identical and $\rho = 5$. Probability of rallying in state 2 is symmetric to this figure. Moreover, conditional choice probability for $D$ is symmetric as well.




(c) Here we $P[p_t = p]$ for $p \in \{HH, HL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats and $\rho = 4$. Moreover, probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$.




(d) Here we $P[p_t = p]$ for $p \in \{HH, HL\}$ on the equilibrium path. Here, both states have same number of parliamentary seats and $\rho = 5$. Moreover, probability of $LH$ is the same as probability of $HL$ and probability for $LL$ is the same as that of $HH$.

Figure 11: Comparative static for $\rho \uparrow$. Each Figure is meant for model where states have equal parliamentary seats. Figure 11a shows conditional choice probability for $R$ when $\rho = 4$. For this case, Figure 11c shows unconditional probability of popularity vector $p \in \{HH, HL\}$. Figures 11b shows conditional choice probabilities for $R$ when $\rho = 5$. For this case, Figure 11d shows unconditional probability of popularity vector $p \in \{HH, HL\}$.

more probable than $LL$. This happens due to rise in conditional choice probabilities for rallying in a state closer to the election. In conclusion, cost of rallies have purely an indirect effect on popularity. It acts through conditional choice probabilities. The reasons behind the non-monotonicity observed in these figures are again similar to the ones discussed before. Therefore, for brevity we omit discussing that.

$\rho \uparrow$ Please refer to Figure 11a and 11b. Similar to the case when there is only one state, candidates rally earlier with a higher $\rho$ (same reasons apply). Moreover, here if a candidate is popular in a state and not popular in the other, he is eager to rally in the state he is not popular. Moreover, he is less eager to rally in the state he is popular at. Eagerness in both states is higher than in the case of lower $\rho$. If he is not popular in both states, then there is eagerness to rally in both states. Due to this rallying in one particular state is suppressed. If he is popular in both states there is less eagerness to rally in any state.

In Figures 11c and 11d we show how unconditional probabilities of popularity evolve over time. The timing of $HH$ becoming more probable than $HL$ is driven by the fact that candidates are now more eager to rally and therefore, $q^{HL}_{HH,t}$ starts increasing for lower values of $t$. Due to this we see that the reversal between $HH$ and $HL$ happens sooner than before. The rise in probability of $HL$ and the fall in probability of $HH$ again happens due to candidates attempt to maintain status quo in later periods [17]. However, the magnitudes of these probabilities are different. Probability of $HL$ doesn't rise as much as it does with lower $\rho$ and probability of $HH$ rises to a higher value than it does with lower $\rho$. This happens because under higher $\rho$, candidates challenge the status quo for longer periods than before. This leads to a higher probability accumulation for either a candidate becoming popular in both states or unpopular in both states. Due to higher $\rho$ decay in probability is lower. Since the number of periods where candidates attempt to maintain the status quo is lower, the rise in probability of $HL$ is suppressed when compared to the case with lower $\rho$.

# 4 Model Extension: Phased Election

In this model, we have $K$ states, $T + 1$ periods and two candidates ($\{M, G\}$). We assume one popularity measure per state that holds information on both candidates. This popularity measure can take two values in the set $\{H, L\}$. Here $H$ is interpreted as '$M$ is more popular than $G$' while $L$ is interpreted as '$G$ is more popular than $M$'. The game is played over first $T$ periods and in each period a sequential move stage game is played. In this stage game the order of play among the candidates is random. A candidate at his turn must choose at most one state out of $K$ states to rally. The decision making stops at period $T$ and in period $T + 1$ every state declares the results. However, the winner of the election in a state is chosen at state specific periods, $\{T_1, T_2, ..T_K\}$, here $T_k$ is the election period for state $k$.

## 4.1 Electoral Pay-offs:

Now we define electoral pay-offs for candidate $R$. Every state $k$ has a number of parliamentary seats $E_k$. For each state $k$ in period $T_k+1$ if popularity $p_{k,T_k+1} = H$, then all parliamentary seats of state $k$ are received by candidate $M$ otherwise $M$ receives none. Therefore, $M$'s total pay-off will be the sum of parliamentary seats he receives from each state. Therefore, his terminal pay-offs are given by:

[17] We have discussed the rationale behind why $HH$ becomes more probable than $HL$ and $LH$ in the previous subsection.

$$V_{M,T+1}(p_{1,T_1+1}, p_{2,T_k+1}, \dots, p_{K,T_K+1}) = \sum_{k=1}^{K} E_k \times \mathbb{1}\Big\{p_{k,T_k+1} = H\Big\} \tag{19}$$

For candidate $G$ his electoral pay-off is defined as $V_{G,T+1} = \sum_k E_k - V_{M,T+1}$. Therefore, if $M$ looses a parliamentary seat then $G$ simultaneously wins that seat. Moreover, by using these pay-offs, one can see that $G$'s preferred popularity level is $L$[18]. Therefore, we are enforcing that if $M$ is popular (i.e. $p_{k,T+1} = H$) then $G$ is unpopular and vice-e-versa. Hence, $G$ wishes to keep $M$'s popularity to $L$ and $M$ wants to keep his popularity at $H$.

## 4.2 Relevant Popularity

Since, elections in every state do not take places in the terminal period, i.e. each state has its own specific election period, we need to define an additional variable. We call $\tilde{p}_{kt}$ as relevant popularity in state $k$ at period $t$. It is defined below as:

$$\tilde{p}_{kt} = \begin{cases} p_{kt} & T_k \geq t \\ p_{k,T_k+1} & T_k < t \end{cases} \tag{20}$$

The reason for defining this variable is to accommodate the fact that politicians only care for their popularity in a state $k$ when the elections have not taken place yet. Once, the elections have taken place the current popularity is irrelevant as the voters have already made their decisions. Now, given equations 1 and 20 we can derive the transition probabilities for relevant popularity, the final expression is given below:

$$P\Big(\tilde{p}_{k,t+1} = H\Big|a_{Mkt}, a_{Gkt}, \tilde{p}_{kt}, t\Big) = \begin{cases} P\Big(\tilde{p}_{k,t+1} = H\Big|a_{Mkt}, a_{Gkt}, \tilde{p}_{kt}\Big) & T_k \geq t \\ 1 & T_k < t, \tilde{p}_{kt} = H \\ 0 & T_k < t, \tilde{p}_{kt} = L \end{cases} \tag{21}$$

Then, we define $\tilde{p}_t = (\tilde{p}_{1t}, \tilde{p}_{2t}, \dots, \tilde{p}_{kt})$ as relevant popularity standing across states at time $t$ and let $p \in \{H, L\}^K$ be an arbitrary popularity standing. We can show that relevant popularity is also uncorrelated with each other. Based, on these results, we derive $P\Big(\tilde{p}_{t+1} = p\Big|a_{Rt}, a_{Dt}, \tilde{p}_t\Big)$[19].

[18] If at period $T_k+1$ in state $k$ popularity was $L$, $M$ will loose all parliamentary seats from $k$ and simultaneously $G$ will win all parliamentary seats

[19] Please note that the following holds.

$$\mathbb{E}\Big[\mathbb{1}\{p_{t+1} = p\}\Big|a_{Rt}, a_{Dt}, p_t\Big] = P\Big(p_{t+1} = p\Big|a_{Rt}, a_{Dt}, p_t\Big)$$
$$\mathbb{E}\Big[\mathbb{1}\{p_{kt+1} = p_k\}\Big|a_{Rt}, a_{Dt}, p_t\Big] = P\Big(p_{kt+1} = p_k\Big|a_{Rt}, a_{Dt}, p_t\Big)$$

Which states: expectation of an indicator function is the probability of the event it indicates.

$$P\Big(\tilde{p}_{t+1} = p\Big|a_{Mt}, a_{Gt}, \tilde{p}_t, t\Big) = \prod_{k=\{1,\ldots,K\}} P\Big(\tilde{p}_{kt+1} = p_k\Big|a_{Mkt}, a_{Gkt}, \tilde{p}_{kt}, t\Big)$$
$$= \prod_{k=\{1,\ldots,K\}} \Big[P\Big(\tilde{p}_{kt+1} = H\Big|a_{Mkt}, a_{Gkt}, \tilde{p}_{kt}, t\Big) \times \mathbb{1}\{p_k = H\} \tag{22}$$
$$+ \Big(1 - P\Big(\tilde{p}_{kt+1} = H\Big|a_{Mkt}, a_{Gkt}, \tilde{p}_{kt}, t\Big)\Big) \times \mathbb{1}\{p_k = L\}\Big]$$

In the above equations, the first equality follows because $p_{kt+1}$ and $p_{k't+1}$ are uncorrelated for $k \neq k'$. Since $p_{kt+1}$ and $p_{k't+1}$ are uncorrelated then $\tilde{p}_{kt+1}$ and $\tilde{p}_{k't+1}$ are also uncorrelated. The second equality follows from expanding $P(\tilde{p}_{kt+1} = p_k|.) = P(\tilde{p}_{kt+1} = H|.) \times 1\{p_k = H\} + (1 - P(\tilde{p}_{kt+1} = H|.)) \times 1\{p_k = H\}$. Now we express electoral pay-offs as a function of relevant popularity. It can be shown that electoral pay-offs are a function of $\tilde{p}_{T+1}$ alone and therefore we do not need to worry about for past popularity as relevant popularity implicitly take account of that.[20]

$$V_{M,T+1}(\tilde{p}_{1,T_1+1}, p_{2,T_k+1}, \ldots, p_{K,T_K+1}) = \sum_{k=1}^{K} E_k \times \mathbb{1}\Big\{\tilde{p}_{k,T+1} = H\Big\} \tag{23}$$

Based on our discussion so far and the two assumptions we can show that best responses can defined as a function of current relevant popularity, cost shocks and, in the case of the second mover, first mover action. The proposition 3.1 lays out the characterization of the equilibrium conditional choice probabilities.. The only change that we have is to replace $p_{kt}$ with $\tilde{p}_{kt}$ everything else stays the same. Same goes for proposition 3.2. This model can be applied to the case of Indian Parliamentary Elections.

# 5 Data

We are using one primary data source for my study. We rely on Indian Express's 2019 election repository[21] as it not only states the rally locations along with time but it also provides video evidence of its occurrence. Rallies by both candidates are recorded and 'streamed'[22] on YouTube on the official channels of the parties. These recordings are available on the party's official YouTube channel and the links can be found on the Indian Express's election data repository. Moreover, in terms of locations Indian Express provides the name of the state, district or parliamentary constituency[23] and the local town names. Having this information allowed us to obtain Longitude and Latitude of these locations.

[20]Since $T + 1 > T_k$ for all $k = 1, 2, \ldots, K$ we have $\tilde{p}_{kT+1} = p_{kT_k+1}$ for all $k = 1, 2, \ldots, K$.

[21]For Narendra Modi, the rallies with date and locations are given Modi Rallies . For Rahul Gandhi the same is given at Gandhi Rallies

[22]Live telecast on youtube is termed as streaming. It doesn't need to be a telecast of an event.

[23]Sometimes these are the same but there are few cases when they are different

We used two data sources to obtain a constituency-wise election schedule. These are our secondary data sources and are useful in constructing the electoral pay-offs of the candidates in our model. The first data source is provided by Firstpost[24] which is an Indian news and media website. Like Indian Express they also maintained an election repository however it less detailed and therefore they do not have any information of rally locations. In order to verify Firstpost's information we used India Today's[25] website. India Today is also another media company which maintains a website, magazine and has a news channel. By combining these two data sources we were able to obtain a constituency wise election schedule[26]. Once, we had the constituency schedule we used `https://www.geonames.org/` to obtain Longitudes and Latitudes of the centroid of each constituency. Once we obtained the centroids of towns where PM candidates went and the centroids of Parliamentary constituencies, we matched the rally locations to constituencies that were geographically closest. We also verified the match by consdering if names of states, and the district or constituency names on the rally locations were matching from the both data sets or not. There were very minor mistakes in the spellings due to which some names would not have matched[27] if one would have considered string matching. Developing a fuzzy string matching for such a small dataset would have been excessive.

Once, we had all the information we also decided to remove the states that did not have total rallies more than 4. There were 11 such states and these states are Andhra Pradesh, Arunachal Pradesh, Chandigarh, Goa, Himachal Pradesh, Jammu and Kashmir and Jharkhand. Apart from Andhra Pradesh and Jharkhand these states have smaller number of constituencies. For instance Goa has only two constituencies. Here, Andhra Pradesh is dominated by regional political party that was not aligned with either alliances. Neither did the candidates rally intensively in these states. As a result we decided to remove this state. Jharkhand has been removed due to not having enough rallies by both candidates combined and therefore it does not suite the purpose of our study.

Once we removed these states and the corresponding rallies, we use the rest of the data for

[24] The election schedule on Firstpost's website can be viewed at `https://www.firstpost.com/india/lok-sabha-election-2019-full-schedule-with-dates-phases-constituency-wise-details-all-you-need-to-know-6411351.html`

[25] The election schedule on India Today's website can be viewed at`https://www.indiatoday.in/elections/lok-sabha-2019/story/lok-sabha-election-2019-dates-full-schedule-constituency-wise-details-all-you-need-to-know-1476069-2019-03-12`

[26] The election schedule provided by Election Commission of India is not at the constituency level.`https://www.elections.in/indian-general-election/2019/`. If you visit the links provided under the state names, you will find that there is no information on election schedule. You can proceed to check constituency web portals, but there is no information regarding the election schedule even there.

[27] For instance Tiruchirapalli' and 'Tiruchirappalli, 'Tiruvallur' and 'Thiruvallur', 'Chhikodi','Chikkodi' are just few examples. Another type of error that occurrs is of the following form 'Autonomous' and 'Autonomous District'. In order to avoid string matching errors we found it easier to use Longitudes and Latitudes and then calculate the geographic distances for matching the two data sources.

our analysis. Similar to US case, here candidates also consider to rally more than once in a given day. Therefore, we divide the days in 4 periods and used similar criteria to assign sub-periods to rallies in a given day. From these assigned sub-periods we backed out the periods which are equivalent to the definition of our model. Since the procedure is similar to the case of US, we omit the details of the criteria for brevity.

In India, once elections begin no polling agency is allowed to release the details of their polls as it can interfere with voter preferences. Due to this fact a rich data on polling which has daily and state-wide variation is not possible. Therefore, we do not posses any data on polls that can used as a supplement for observing popularity. Our inference on how popularity would evolve can only come from the candidate actions and not on polls themselves. We are still working on understanding that how can one use only candidate actions in order to understand the evolution of popularity.

# 6 Calibration

In this section, we calibrate the model for India and see if it is able to match certain moments in the data. For this exercise, we consider a model with 7 states. Each state refers to the group of constituencies belonging to a particular Phase. For instance state 1 represents Phase $I$ constituencies, state 2 represents Phase $II$ constituencies and so on. Therefore, we will be referring to Phases $\{I, II, \ldots, VII\}$ as states $\{1, 2, \ldots, 7\}$. Since each day is divided into 4 periods, we have a total of $T = 75 * 4$ periods. Please refer to table 2 to know the parameter values that we consider to simulate the model. For these parameter values we calculate equilibrium quantities defined in propositions 3.1 and 3.2. We use these quantities to calculate predictions on the moments that can be calculated in the sample. We discuss these moments in the following set of paragraphs.

First we need to derive conditional choice probability of rallying in a state $k$ prior to observing the current relevant popularity. For this purpose we need to evaluate the unconditional probability of realizing current relevant popularity $\tilde{p}_t$. These probabilities (for each $t = 1, 2, \ldots, T$) can be evaluated recursively. Please refer to proposition 6.1.

**Proposition 6.1** *Normalizing* $\tilde{f}_1(\tilde{p}_1) = \frac{1}{2^K}$, $f_t(\tilde{p}_t)$ *for each* $t = 2, 3, \ldots, T$ *is given by the following:*

$$\tilde{f}_t(p) = \sum_{k,l \in \{0,1,\ldots,K\}} \sum_{p' \in \{H,L\}^K} P\Big[p \Big| a_{t-1} = (k,l), p', t\Big] \times \sigma_t\Big(a_{t-1} = (k,l); \tilde{p}_{t-1} = p'\Big) \times \tilde{f}_{t-1}(p') \qquad (24)$$

Given the propositions 6.1 and 3.2 it is possible for us to derive conditional choice probability of rallying in a state $k$ prior to observing the current relevant popularity. We denote these probabilities by $\sigma_{Mt}(k)$ and $\sigma_{Gt}(l)$ for $M$ and $G$ respectively. These probabilities are calculated as:

$$\begin{aligned}\sigma_{Mt}(k) &= \sum_{l\in\{0,1,\ldots,K\}} \sum_{p\in\{H,L\}^K} \sigma_t(a_{Mt}=k, a_{Gt}=l, \tilde{p}_t=p) \times \tilde{f}_t(p) \\ \sigma_{Gt}(l) &= \sum_{k\in\{0,1,\ldots,K\}} \sum_{p\in\{H,L\}^K} \sigma_t(a_{Mt}=k, a_{Gt}=l, \tilde{p}_t=p) \times \tilde{f}_t(p)\end{aligned} \tag{25}$$

Now we can use $\sigma_{Mt}(.)$ and $\sigma_{Gt}(.)$ to test if the model is capable of matching key moments from the data. The first set of moments we will compare are going to be probability of rallying in any state across time. For this exercise, we divide the data into 5 day bins (20 period bins). For each bin we estimate probability of rally and its corresponding confidence interval. These estimates are calculated as followed:

1 First we construct 20 period bins (equivalently 5 day bins). We denote these bins as $b_1, b_2, \ldots, b_{15}$. Where $b_d$ is defined as:

$$b_d = \Big\{t : t \in \Big\{(d-1)\times 20+1, (d-1)\times 20+2, \ldots, d\times 20\Big\}\Big\} \tag{26}$$

Moreover, let $b(t) = \{b_d : t \in b_d\}$.

2 For each bin we calculate the probability of rallying as:

$$P^{\text{obs}}_{id} = \frac{1}{|b_d|}\sum_{t\in b_d} \mathbb{1}\{A_{it} \neq 0\} \tag{27}$$

Where $A_{it}$ is a multinomial variable that can take up to 8 values. It is defined below as:

$$A_{it} = \begin{cases} 0 & \text{if } i \text{ chose not to rally} \\ k & \text{if } i \text{ chose to rally in Phase } k \text{ constituency} \end{cases} \tag{28}$$

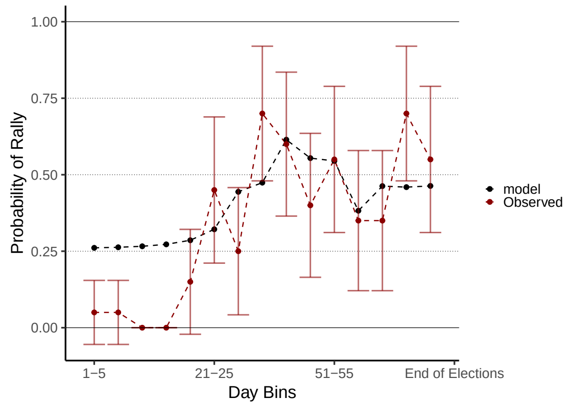
3 We calculate standard error for each bin by treating $A_{it}$ as Bernoulli random variable as followed:

$$s^{\text{obs}}_{id} = \sqrt{\frac{P^{\text{obs}}_{id} \times (1 - P^{\text{obs}}_{id})}{|b_d|}} \tag{29}$$

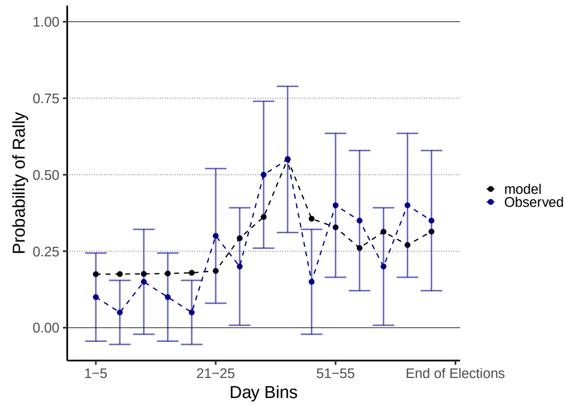
4 Confidence intervals are calculated using $t$-distribution with $|b_d| - 1$ degrees of freedom.

We can use $\sigma_{Mt}(.)$ and $\sigma_{Gt}(.)$ to calculate model's prediction for $P^{\text{obs}}_{id}$. For this purpose first we calculate probability of not rallying for each candidate and then consider its complement probability. We use the average of these probability for each period bins. The expression is given below

$$P^{\text{model}}_{id} = \frac{1}{|b_d|}\sum_{t\in b_d}\Big(1 - \sigma_{it}(0)\Big) \tag{30}$$




(a) $(1-\sigma_{Mb}(0))$ vs $P^{\text{obs}}_{Mb}$: Here we compare $M$'s calibrated probability of rallies with $M$'s observed probability of rallies. Each point on the $x-$axis represents a 20-period bin (or 5-day bin). On the $y-$axis we have probability of rallying. Here $M$'s calibration doesn't fit well in the initial periods as there are very few rallies. However from the fifth bin onwards ($64^{\text{th}}$) the fit improves. The model under predicts the second $14^{\text{th}}$ bin probability.




(b) $(1-\sigma_{Gb}(0))$ vs $P^{\text{obs}}_{Gb}$: Here we compare $G$'s calibrated probability of rallies with $G$'s observed probability of rallies. Each point on the $x-$axis represents a 20-period bin (or 5-day bin). On the $y-$axis we have probability of rallying. Here $G$'s calibrated probabilities are closer to the observed probabilities than $M$'s probabilities.

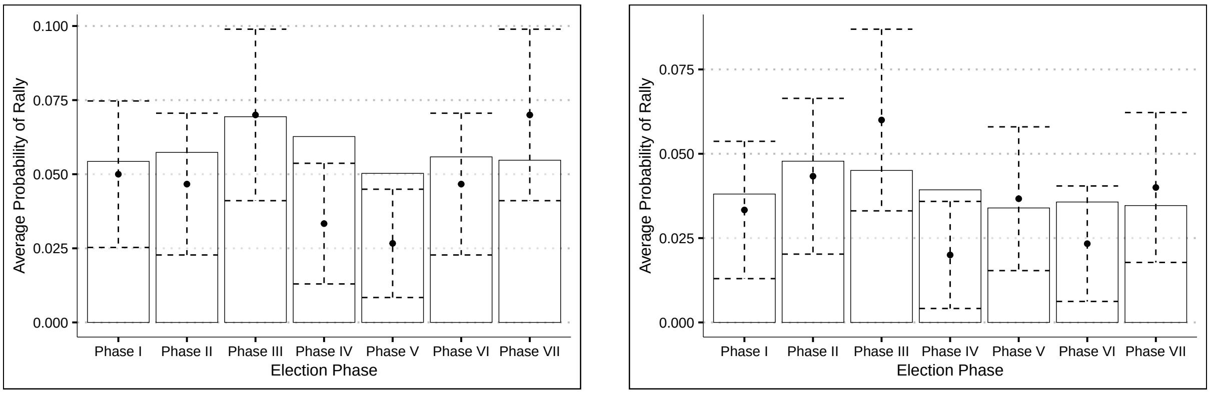
Figure 12: Here, we are plotting estimates of probability of rallying for 5 day (20 period) bins along with its confidence intervals. We label these estimates as Observed as they are have been estimated from the observed sample. We treat the event of rallying as a Bernoulli random variable and therefore calculate the confidence intervals accordingly. We also plot average of $\sigma_{it}(k)$ for each bins and label them as model as these values have been obtained by calibrating the model. Panel (a) shows the estimates and the calibrated values for Narendra Modi's ($M$) strategies. Panel (b) shows the estimates and the calibrated values for Rahul Gandhi's ($G$) strategies.

Consider the Figures 12a and 12b. Here we are comparing $P^{\text{model}}_{id}$ with $P^{\text{obs}}_{id}$. Firstly, unlike US $P^{\text{obs}}_{id}$ has a non-linear pattern. This pattern is explained by the very fact that elections in different constituencies happen at different dates. By introducing this feature in the model we are also able to support this pattern in the model's equilibrium strategies. It is evident by looking at how $P^{\text{model}}_{id}$. The fit for Gandhi's decisions (Figure 12b) are much more superior than the fit for Modi's decisions (Figure 12a). The reason behind this bad fit in case of Modi's decision is a period of eighteen days when Narendra Modi did not conduct many rallies. This period coincides with 2019 India–Pakistan border skirmishes which began on 14th February and lasted till 22 March. Due to this national security issue, I suspect Narendra Modi had to prioritize office duties over campaigning. Additionally, there are anecdotal evidences in national media pointing out that Election Commission was considering delaying the elections due to the rising tension with Pakistan. However, the peace offer by India was accepted by Pakistan on 22nd March and a week later one can observe that Narendra Modi begins rallying intensively. This is the point where our model fit improves significantly and most calibrated values are not statistically different from the estimated values.

Our second moment of interest is going to be cross sectional in nature. For this we consider average probability of rallying in a constituency belonging to Phase $k$. This can be calculated from the data by using the following estimator:

$$P^{obs}_{ik} = \frac{1}{T} \sum_{t=\{1,2,\ldots,T\}} \mathbb{1}\{A_{it} = k\} \tag{31}$$

Given that $A_{it}$ is a multinomial random variable, the random variable $\mathbb{1}\{A_{it} = k\}$ is a binomial random variable and therefore we can use the standard formulae for computing its standard deviation. Given that, the standard error for these quantities can be calculated as




(a) $\frac{1}{T}\sum_t \sigma_{Mt}(k)$ v.s. $\frac{1}{T}\sum_t A_{Mkt}$. Here we compare model's mean probability of rallies in each Phase with estimated mean probability of rallies in each Phase. The bars in the figure represent model's predictions. The points represent the point estimates of mean probability of rallying in a state. The confidence intervals are calculated by treating $\frac{1}{T}\sum_t A_{Mkt}$ as a binomial random variable. Here 5 out of 7 model predicted moments are not statistically different from estimated moments.

(b) $\frac{1}{T}\sum_t \sigma_{Gt}(k)$ v.s. $\frac{1}{T}\sum_t A_{Gkt}$. Here we compare model's mean probability of rallies in each Phase with estimated mean probability of rallies in each Phase. The bars in the figure represent model's predictions. The points represent the point estimates of mean probability of rallying in a state. The confidence intervals are calculated by treating $\frac{1}{T}\sum_t A_{Gkt}$ as a binomial random variable. Here 6 out of 7 model predicted moments are not statistically different from estimated moments.

Figure 13: Here we are plotting estimates of mean probability of rallying in a Phase along with its confidence intervals. We show these point estimates and the confidence intervals in the figure. We also plot model predictions for these moments obtained from the calibration exercise. These predictions are plotted as bars in the figure. Panel 13a and 13b shows the estimates and calibrated values for Narendra Modi ($M$) and Rahul Gandhi's $G$ strategies respectively.

followed:

$$s_{ik}^{obs} = \sqrt{\frac{P_{ik}^{obs} \times (1 - P_{ik}^{obs})}{T-1}} \tag{32}$$

For the confidence intervals we use the $t-$distribution with $T-1$ degrees of freedom. The model's prediction of these moments can be calculated by considering $\sigma_{Mt}(.)$ and $\sigma_{Gt}(.)$ and its given as:

$$P_{ik}^{\text{model}} = \frac{1}{T} \sum_{t=\{1,2,\ldots,T\}} \sigma_{it}(k) \tag{33}$$

Consider the Figures 13a and 13b. Here we compare $P_{ik}^{\text{model}}$ and $P_{ik}^{\text{obs}}$. Firstly, it is important to note that $P_{\text{obs}}^{ik}$ and $P_{\text{model}}^{ik}$ are not perfectly proportional to $E_k$. This arises due to the differential election periods for each Phase $k$. Most of our model predictions are not statistically different from estimates, only 3 out of 14 are statistically different. All moments are not easy to explain just on the basis of 5 parameters. In the model we force each state or Phase $k$ to follow identical popularity process. However, in a country like India such an assumption is strong. Specially when demographics (being an important predictor of political preferences) are not being accounted. For instance, in Phase $VII$ there are more Hindi speaking constituencies that are having an election. In these constituencies Modi can have higher returns to rallies than other Phase constituencies. As a result he will have a higher incentive to rally in Phase $VII$ constituencies. At this moment we do not posses a simpler specification that can use demographics without increasing the parametric burden to a great extent. Therefore, we keep these aspects out of the model. However, despite not accounting for these differential popularity processes across Phases, our cross-sectional fit is still appreciable as 11 out of 14 cross-sectional moments are not statistically different from model predictions. Therefore, extending the model to allow for demographic dependent popular-

Table 2: Calibration Exercise

| **Panel A: Parameter Specification** | | | | | | | |
|---|---|---|---|---|---|---|---|
| Parameter | $c_M$ | $c_G$ | $\alpha_M$ | $\alpha_G$ | $\rho$ | $K$ | $T$ |
| Values | 3 | 3.5 | 4 | $-3.6$ | 5 | 7 | 300 |

| **Panel B: Phase Level Values** | | | | | | | |
|---|---|---|---|---|---|---|---|
| Phase (k) | *I* | *II* | *III* | *IV* | *V* | *VI* | *VII* |
| Seats ($E_k$) | 47 | 93 | 114 | 68 | 45 | 54 | 51 |
| Election Period ($T_k + 1$) | 149 | 177 | 197 | 221 | 249 | 273 | 301 |

| **Panel C: Mean Squared Errors** | | |
|---|---|---|
| | Model CCPs $s_{it}(k) = \sigma_{it}(k)$ | Phase wise Average $s_{it}(k) = \frac{1}{T}\sum_{t=1} A_{ikt}$ |
| Mean Square Error for $M$ as given by: $\text{MSE}_M = \frac{1}{T}\sum_{t=1}^{T}\sum_{k=0}^{7}\left(A_{Mkt} - s_{Mt}(k)\right)^2$ | 0.481 | 0.552 |
| Mean Square Error for $G$ as given by: $\text{MSE}_G = \frac{1}{T}\sum_{t=1}^{T}\sum_{k=0}^{7}\left(A_{Gkt} - s_{Gt}(k)\right)^2$ | 0.416 | 0.438 |
| Total Mean Square Error calculated as: $\text{MSE} = \frac{1}{T}\sum_{t=1}^{T}\sum_{k=0}^{7}\sum_{i\in\{M,G\}}\left(A_{ikt} - s_{it}(k)\right)^2$ | 0.896 | 0.99 |

ity will only improve the model even further.

We also calculate the mean squared error of the whole fit. In order to do so, we consider the random variable, $\tilde{A}_{it} = (A_{i0t}, A_{i1t}, \ldots, A_{i7t})$. Here $A_{ikt}$ is an indicator function which takes value 1 if $A_{it} = k$[28]. Now given, we can calculate the mean squared errors for the calibrated model as followed:

[28] If $i$ chose Phase or State $k$ or chose not to rally ($k = 0$).

$$
\begin{aligned}
\text{MSE} &= \frac{1}{T}\sum_{t=1}^{T}\sum_{k=0}^{7}\sum_{i\in\{M,G\}}\Big(A_{ikt}-\sigma_{it}(k)\Big)^2 \\
\text{MSE}_i &= \frac{1}{T}\sum_{t=1}^{T}\sum_{k=0}^{7}\Big(A_{ikt}-\sigma_{it}(k)\Big)^2
\end{aligned}
\tag{34}
$$

Here, MSE will consider the fit of the whole model while $\text{MSE}_i$ will consider the fit of individual candidates. We will compare the mean squared errors of the model with the standard deviation of $A_{it}$ with respect to Phase wise averages[29]. The results from this exercise are given in Panel C of Table 2.

# 7 Conclusion

The paper constructs a model of dynamic electoral competition where politicians compete against each other to stay popular on the election day. The model possesses a finite time horizon and a perfect information structure. The combination of these features is sufficient for applying backward induction to compute equilibrium conditional choice probabilities, which are unique. This contribution combines three important features— (i) finite time horizon, (ii) dynamic electoral competition and (iii) unique equilibrium. These three features have been a trilemma in the literature on electoral games (Kawai and Sunada, 2022; Erikson and Palfrey, 2000; de Roos and Sarafidis, 2018; Meirowitz, 2008; Polborn and Yi, 2006; Gul and Pesendorfer, 2012; Strömberg, 2008). This formulate the main contribution of the paper.

I exploit the model to come up with certain estimates for Modi Rallies. This formulates one of the first examination of Modi rallies. I also find that candidates have high effectiveness in India. Political rallies in India are more than capable of reshaping the popularity of a candidate. These political rallies can strongly influence the popularity path. It points to the possibility that political rallies can influence electoral outcomes in India. It is evident from the data that candidates use political rallies in the majority of states. Not all of these states are "*swing states*" and therefore candidates are also visiting states where there opponent has strong holding. To fully test how sensitive electoral outcomes can be to political rallies, we plan to conduct a counter-factual experiment. In the full version of the model, it is possible to run simulations in a scenario where rallies are absent. One can calculate the probability of winning in the absence of rallies and compare it with the case where rallies are allowed. The difference in probability of winning can give us the degree of how sensitive electoral outcomes are to political rallies.

[29] $\frac{1}{T}\sum_t A_{ikt}$

We provide the very first estimates for the cost of a rally. Our cost estimates are inversely proportional to the willingness of a candidate to hold a political rally. Therefore, this calibration exercise helps us to find an average cost for political rallies held by Narendra Modi and Rahul Gandhi. However, we do not have a parliamentary seat to Indian Rupee conversion rate estimates. The conversion rate of a parliamentary seat can not be a small value. These seats also carry legislative powers in addition to executive power. Therefore these seats are supposed to be of high importance and value.

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