UG Empirical Industrial Organization

Lecture 9: Models of Competition in Prices and Quantities

Anubhav Jha[1]
Ashoka University

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[1] This course is based on Victor Aguirregabiria's Empirical IO book. The slides (in my first year of teaching) are extremely close to his slides. These will change in the future iterations of this course.

OUTLINE OF TODAY's LECTURE

1. Introduction
2. Estimating Marginal Costs given a form of competition
    2.1 Perfect competition
    2.2 Cournot competition
    2.3 Bertrand competition: differentiated products

Introduction

- Firms' decisions on how much to produce and what price to charge are fundamental determinants of firms' profits.
- These decisions involve strategic competition between firms.
  - **Market for homogeneous goods:** If a firm increases its quantity, price declines and this reduces profits of all the other firms in the market.
  - **Market for differentiated products:** If a firm reduces its own price, the quantity demanded of other products declines, reducing the profits of other firms.
- These strategic interactions have first-order importance for understanding competition and outcomes in most industries.
- For this reason, models of competition where firms choose prices or quantities are at the core of Industrial Organization.

Equilibrium Model of Competition

- The answer to many economic questions requires not only the estimation of demand and cost functions, but also an understanding of how firms compete with each other.
- For instance, suppose we are interested in measuring the effects of:
  - An increase in sales tax.
  - A rise in input prices.
  - Entry of a new firm or product into the market.
  - A merger between two firms.
  - ...
- The answers to these questions can vary greatly depending on whether the market structure is:
  - Perfect Competition,
  - Oligopoly Competition, or
  - Collusion.

Empirical Models of Price or Quantity Competition

- To answer these empirical questions, we need to know or estimate:
  - Firms' marginal costs (**Topic 1** and **Topic 3**)
  - Demand system (**Topic 2**)
  - Form of competition (**Topic 3**)
- We can distinguish three classes of applications of empirical models of competition in prices or quantities:
  1. Estimation of firms' marginal costs.
  2. Identification of the form of competition.
  3. Joint identification of marginal costs and form of competition.

Estimation of Firms' Marginal Costs

- In many empirical applications, researchers have data on firms' prices and quantities, but no data on firms' costs.
- In this context, empirical models of competition in prices or quantities provide an approach to estimate firms' marginal costs and the structure of these costs.
- **Key idea:** Profit maximization implies the condition

$$\text{MR} = \text{MC}$$

  where the precise definition of marginal revenue (MR) depends on the form of competition — e.g., Perfect Competition, Cournot, Bertrand, Stackelberg, or Collusion.
- Given an estimated demand system and an assumption about the form of competition, we can construct estimates of firms' marginal revenues (MRs).
- Then, using the equilibrium condition $\text{MC} = \text{MR}$, we obtain estimates of firms' marginal costs (MCs).

Identification of the Form of Competition

- Suppose the researcher has data on:
  - Prices and quantities — to estimate the demand system.
  - Output, inputs, and input prices — to estimate the production function and marginal costs.
- Given an assumption about the Form of Competition (e.g., Perfect Competition, Cournot, Collusion), the researcher can use the approach from the previous slide to obtain an independent estimate of firms' marginal costs.
- The researcher can then test whether the implied marginal costs under a given competitive structure are consistent with those estimated from the production function.
- This allows identification of the Form of Competition that best fits the data — for instance, providing evidence of possible firms' collusion.

Joint Identification of MCs & Form of Competition

- Suppose the researcher has data on prices and quantities to estimate the demand system, but no data on inputs or input prices to directly estimate firms' marginal costs.
- Given the estimated demand, the equilibrium condition

$$\text{Marginal Cost} = \text{Marginal Revenue}$$

  can be used to simultaneously identify or estimate both the marginal costs (MCs) and the form of competition.
- This joint estimation approach is known as the **Conjectural Variation Approach**.

Some Seminal References in This Literature

- Bresnahan, T. (1982): "The Oligopoly Solution Concept is Identified," *Economics Letters*, 10, 87–92.
- Bresnahan, T. (1987): "Competition and Collusion in the American Automobile Market: The 1955 Price War," *Journal of Industrial Economics*, 35, 457–482.
- Genesove, D. and W. P. Mullin (1998): "Testing Static Oligopoly Models: Conduct and Cost in the Sugar Industry, 1890–1914," *The RAND Journal of Economics*, 29(2), 355–377.
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Estimating Marginal Costs

Given the Form of Competition:

Case 1: Perfect Competition

Estimating MCs: Perfect Competition

- We first illustrate this approach in the context of a perfectly competitive industry for a homogeneous product.
- The researcher has data on the market price and on firms' output for T periods of time (or geographic markets):

$$\{p_t, q_{it} : i = 1, 2, \ldots, N_t \text{ and } t = 1, 2, \ldots, T\}$$

- N_t is the number of firms active in period t. The profit of firm i is given by:

$$\Pi_{it} = p_t q_{it} - C_i(q_{it})$$

- Under perfect competition, the marginal revenue of any firm i equals the market price p_t.
- Profit maximization implies:

$$p_t = MC_i(q_{it}) \quad \text{for every firm } i,$$

where $MC_{it} \equiv C_i'(q_{it})$.

Estimating MCs: Perfect Competition [2/6]

- Suppose that marginal costs are given by:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp\{\varepsilon_{it}^{MC}\}$$

  where θ is a technological parameter and ε_{it}^{MC} is an unobservable capturing the cost efficiency of firm i.
- Interpretation of θ:
  - (i) Constant marginal cost: $\theta = 0$
  - (ii) Increasing marginal cost: $\theta > 0$
  - (iii) Decreasing marginal cost: $\theta < 0$
- Using the equilibrium condition $p_t = MC_i(q_{it})$, we can estimate θ and the cost efficiency term ε_{it}^{MC} for each firm i.

Estimating MCs: Perfect Competition [3/6]

- The equilibrium condition

$$p_t = MC_i(q_{it})$$

implies the following regression model in logarithms:

$$\ln(p_t) = \theta \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Using data on prices and quantities, we can estimate the slope parameter θ in this regression equation.
- Given an estimate of θ, we can compute the cost efficiency term as the residual:

$$\varepsilon_{it}^{MC} = \ln(p_t) - \theta \ln(q_{it})$$

- Therefore, we can estimate each firm's marginal cost function as:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp\{\varepsilon_{it}^{MC}\}$$

Estimating MCs: Perfect Competition [4/6]

- Recall the estimating equation:

$$\ln(p_t) = \theta \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Estimation of this equation by **OLS** suffers from an **endogeneity problem**.
- The equilibrium condition implies that less efficient firms (with larger ε_{it}^{MC}) produce a lower level of output.
- Therefore, the regressor $\ln(q_{it})$ is **negatively correlated** with the error term ε_{it}^{MC}.
- This negative correlation causes the OLS estimator to be **downward biased**.
- As a result, the OLS estimate may indicate **increasing returns to scale (IRS)** — i.e., $\theta < 0$ — even when the true technology exhibits **decreasing returns to scale (DRS)**, i.e., $\theta > 0$.

Estimating MCs: Perfect Competition [5/6]

- The endogeneity problem does not disappear even if we consider the model in **market means**:

$$\ln(p_t) = \theta \ln(q_t) + \varepsilon_t^{MC}$$

  where $\ln(q_t)$ and ε_t^{MC} represent the mean values of $\ln(q_{it})$ and ε_{it}^{MC} over all firms active in period t.
- We still have that $\ln(q_t)$ and ε_t^{MC} are **negatively correlated**.
- Intuitively:
  - In time periods with larger aggregate cost shocks (ε_t^{MC}),
  - There is lower average log-output ($\ln(q_t)$).

Estimating MCs: Perfect Competition [6/6]

- Recall the market-level estimating equation:

$$\ln(p_t) = \theta \ln(q_t) + \varepsilon_t^{MC}$$

- We can address the **endogeneity problem** by using **instrumental variables (IV)**.
- Suppose that X_t^D is a vector of observable variables that affect demand but not marginal costs (MCs).
- These demand variables should be:
  - **Correlated** with $\ln(q_t)$ — because demand shocks influence firms' output decisions, and
  - **Uncorrelated** with ε_t^{MC} — because they do not directly affect firms' cost conditions.
- Under these assumptions, the variables X_t^D can be used as valid instruments for $\ln(q_t)$, allowing the **consistent estimation of** θ.

Estimating Marginal Costs

Given the Form of Competition:

Case 2: Cournot Competition

Estimating MCs: Cournot Competition

- We continue to consider a **homogeneous product industry**.
- The researcher observes data on quantities and prices over T periods of time:

$$\{p_t, q_{it} : i = 1, 2, \ldots, N_t \text{ and } t = 1, 2, \ldots, T\}$$

- Unlike perfect competition, we now assume that firms compete in **Nash–Cournot** environment.
- The profit of firm i is given by:

$$\Pi_{it} = p_t q_{it} - C_i(q_{it})$$

- The demand side can be represented by the **inverse demand function**:

$$p_t = P(Q_t, X_t^D)$$

  where:
  - $Q_t \equiv \sum_{i=1}^{N_t} q_{it}$ is the total market output, and
  - X_t^D is a vector of exogenous market characteristics that affect demand.

Estimating MCs: Cournot Competition [2/9]

- Each firm chooses its own output q_{it} to maximize profit.
- Since profit equals revenue minus cost, profit maximization implies:

$$\text{Marginal Revenue (MR)} = \text{Marginal Cost (MC)}.$$

- The marginal revenue function is:

$$MR_{it} = \frac{d\,\text{Revenue}_{it}}{dq_{it}} = p_t + \frac{dp_t}{dq_{it}} q_{it}$$

- Using the inverse demand function $p_t = P(Q_t, X_t^D)$ and the fact that $Q_t = q_{it} + Q_{(-i)t}$, we obtain:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\left(1 + \frac{dQ_{(-i)t}}{dq_{it}}\right) q_{it}$$

- where:
  - $P'_Q(Q_t, X_t^D)$ = derivative of the inverse demand curve with respect to total output Q_t, and
  - $Q_{(-i)t}$ = total output of all firms other than firm i.

Estimating MCs: Cournot Competition [3/9]

- Recall the marginal revenue function:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\left(1 + \frac{dQ_{(-i)t}}{dq_{it}}\right) q_{it}$$

- The term $\frac{dQ_{(-i)t}}{dq_{it}}$ represents the **belief (or conjecture)** that firm i has about how other firms will adjust their outputs when this firm marginally changes its own output.
- Under the assumption of **Nash–Cournot competition**, this conjecture is zero:

$$\text{Nash–Cournot:} \quad \frac{dQ_{(-i)t}}{dq_{it}} = 0$$

- In other words, each firm takes as fixed the total quantity produced by the rest of the firms, $Q_{(-i)t}$, and chooses its own output q_{it} to maximize profit.

Estimating MCs: Cournot Competition [4/9]

- Therefore, the first-order condition of optimality under **Nash–Cournot competition** is:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\, q_{it} = MC_i(q_{it})$$

- Since $P'_Q(Q_t, X_t^D) < 0$ (because the demand curve is downward sloping), it follows that:

$$MR_{it} < p_t$$

- If the marginal cost $MC_i(q_{it})$ is a non-decreasing function, then the optimal output under Cournot competition satisfies:

$$q_{it}^{\text{Cournot}} < q_{it}^{\text{Perfect Competition}}$$

- **Conclusion:** Oligopoly (Cournot) competition leads to **lower output** and **higher prices** relative to perfect competition.

Estimating MCs: Cournot Competition [5/9]

- Consider the same specification of the cost function as before:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp\{\varepsilon_{it}^{MC}\}$$

- Suppose that the demand function has been estimated in a first step, so that we have a consistent estimate of $P(Q_t, X_t^D)$.
- The researcher can then construct consistent estimates of firms' marginal revenues as:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\, q_{it}$$

- The econometric model can therefore be written as the following regression equation in logarithms:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + \varepsilon_{it}^{MC}$$

Estimating MCs: Cournot Competition [6/9]

- Recall the estimating equation:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Estimation of this regression by **OLS** suffers from the same **endogeneity problem** as in the perfect competition case.
- To address this issue, we can use **instrumental variables (IV)** estimation.
- As in the perfect competition case, we can use observable variables that affect demand but not costs — denoted by X_t^D — as instruments.
- Under **Cournot competition**, however, we can have **additional valid instruments** due to the strategic interaction among firms.

Estimating MCs: Cournot Competition [7/9]

- Suppose the researcher also observes some **exogenous firm characteristics** that affect marginal costs.
- Examples include information on:
  - Firm's wage rate,
  - Capital stock, or
  - Installed production capacity.
- Let these variables be represented by the vector Z_{it}.
- The marginal cost function can then be written as:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp\{Z_{it}\gamma + \varepsilon_{it}^{MC}\},$$

  where γ is a vector of parameters.
- The marginal condition of optimality, in logarithms, becomes:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + Z_{it}\gamma + \varepsilon_{it}^{MC}$$

Estimating MCs: Cournot Competition [8/9]

- Recall the estimating equation:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + Z_{it}\gamma + \varepsilon_{it}^{MC}$$

- Note that the characteristics Z_{jt} of firms $j \neq i$ affect the equilibrium output of firm i.
- Intuition:
  - A smaller Z_{jt} implies that firm j is more cost-efficient.
  - This increases its output, which lowers the market price p_t and thus the marginal revenue MR_{it}.
  - Consequently, the output q_{it} of other firms (those $i \neq j$) becomes smaller.
- Under the assumption that firm characteristics are **exogenous**, i.e.,

$$E(Z_{jt}\, \varepsilon_{it}^{MC}) = 0 \quad \text{for all } (i, j),$$

the characteristics Z_{jt} of other firms can be used as **instrumental variables**.

Estimating MCs: Cournot Competition [9/9]

- For instance, we can use the characteristics of other firms,

$$\sum_{j \neq i} Z_{jt},$$

  as **instrumental variables**.
- Then, the parameters θ and γ can be estimated using the following **moment conditions**:

$$E\left[\left(\frac{Z_{it}}{\sum_{j \neq i} Z_{jt}}\right)\left(\ln(MR_{it}) - \theta \ln(q_{it}) - Z_{it}\gamma\right)\right] = 0$$

- Equivalently, the model can be estimated using a **Two-Stage Least Squares (2SLS)** estimator.

2. Conjectural variation model:

Homogeneous product markets

Estimating MCs: Bertrand with Differentiated Products

- Consider an industry producing **differentiated products**.
- The researcher observes data on prices, quantities, and product characteristics for J products in the industry, where J is large:

$$\{p_i, q_i, X_i\} \quad \text{for } i = 1, 2, \ldots, J.$$

- For simplicity, assume that:
  - Each product is produced by only one firm, and
  - Each firm produces only one product.
- The profit of firm i is given by:

$$\Pi_i = p_i q_i - C_i(q_i)$$

Estimating MCs: Bertrand with Differentiated Products [2/7]

- The demand system arises from a **discrete choice model of demand**:

$$q_i = Hs_i = H\,\sigma_i(p, X)$$

  where:
  - H is the total number of consumers in the market,
  - $s_i \equiv q_i/H$ is the market share of product i, and
  - $\sigma_i(p, X)$ is the market share function implied by the demand model.
- The vectors p and X denote, respectively, prices and product characteristics across all products.
- For instance, under a **logit demand system**, the market share is:

$$\sigma_i(p, X) = \frac{\exp\{-\alpha p_i + X_i\beta + \xi_i\}}{1 + \sum_{j=1}^{J} \exp\{-\alpha p_j + X_j\beta + \xi_j\}}$$

Estimating MCs: Bertrand with Differentiated Products [3/7]

- Under **Bertrand competition**, each firm chooses its price p_i to maximize profit.
- The first-order condition of optimality implies:

$$\frac{d\Pi_i}{dp_i} = 0 \quad \Leftrightarrow \quad \frac{d(p_i q_i)}{dp_i} = \frac{dC_i(q_i)}{dp_i}$$

- Note that profit Π_i depends on p_i both directly and indirectly through q_i.
- Therefore:

$$\frac{d(p_i q_i)}{dp_i} = q_i + p_i \cdot \frac{dq_i}{dp_i}$$

- and

$$\frac{dC_i(q_i)}{dp_i} = MC_i(q_i) \cdot \frac{dq_i}{dp_i}$$

Estimating MCs: Bertrand with Differentiated Products [4/7]

- Combining the two equations,

$$\frac{d(p_i q_i)}{dp_i} = \frac{dC_i(q_i)}{dp_i},$$

we obtain the marginal revenue condition:

$$MR_i = p_i + q_i \cdot \left[\frac{dq_i}{dp_i}\right]^{-1} = MC_i(q_i)$$

- Since $q_i = Hs_i = H\sigma_i(p, X)$, we can write:

$$MR_i = p_i + s_i \cdot \left[\frac{d\sigma_i}{dp_i}\right]^{-1} = MC_i(q_i)$$

- The term $s_i \cdot \left[\frac{d\sigma_i}{dp_i}\right]^{-1}$ is negative. Therefore, $-(s_i \cdot \left[\frac{d\sigma_i}{dp_i}\right]^{-1})$ represents the **price–cost margin**:

$$p_i - MC_i(q_i)$$

- For example, under a **Logit demand system**:

$$\frac{d\sigma_i}{dp_i} = -\alpha s_i(1 - s_i) \quad \Rightarrow \quad p_i - \frac{1}{\alpha(1 - s_i)} = MC_i(q_i)$$

Estimating MCs: Bertrand with Differentiated Products [5/7]

- In general for the Bertrand case, the marginal revenue is given by:

$$MR_i = p_i + s_i \cdot \left[\frac{d\sigma_i}{dp_i} \right]^{-1}$$

- The marginal revenue MR_i depends only on:
  - The product's own price p_i,
  - Its market share s_i, and
  - The estimated demand function $\sigma_i(p, X)$.
- After estimating the demand function, the researcher can compute (or consistently estimate) the marginal revenues MR_i for every firm or product in the market.

Estimating MCs: Bertrand with Differentiated Products [6/7]

- Suppose that the marginal cost function is given by:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp\{X_{it}\gamma + \varepsilon_{it}^{MC}\}$$

- The marginal cost of producing a product depends on the **characteristics of the product**, represented by X_{it}.
- Suppose that the demand function has been estimated in a first step, providing a consistent estimate of $\sigma_i(p, X)$ and hence of MR_{it}.
- Then, the econometric model can be expressed as:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + X_{it}\gamma + \varepsilon_{it}^{MC}$$

Estimating MCs: Bertrand with Differentiated Products [7/7]

- Recall the estimating equation:

$$\ln(MR_{it}) = \theta \ln(q_{it}) + X_{it}\gamma + \varepsilon_{it}^{MC}$$

- Estimation of this regression by **OLS** suffers from the same **endogeneity problem** as in the Perfect Competition or Cournot cases.
- To address this problem, we can use **instrumental variables (IV)**.
- Specifically, we can use the **characteristics of products other than** i, denoted X_{jt} for $j \neq i$, as instruments.
- The corresponding moment conditions are:

$$E\left[\begin{pmatrix} X_{it} \\ \sum_{j \neq i} X_{jt} \end{pmatrix} \left(\ln(MR_{it}) - \theta \ln(q_{it}) - X_{it}\gamma\right)\right] = 0$$