UG Empirical Industrial Organization

Lecture 10: Models of Competition in Prices and Quantities

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[1]This course is based on Victor Aguirregabiria's Empirical IO book. The slides (in my first year of teaching) are extremely close to his slides. These will change in the future iterations of this course.

Outline of Today's Lecture

OUTLINE OF TODAY'S LECTURE

1. Introduction
2. Estimating the form of competition when marginal costs (MCs) are observed
3. Estimating the form of competition without data on marginal costs (MCs)

Introduction

- In the previous lecture, we saw how, given an estimated demand system and an assumption about competition, we can obtain (or estimate) firms' marginal costs.
- In today's lecture, we will see how, given a demand system and firms' marginal costs, we can identify the **form of competition** in a market.
- More specifically, we can identify firms' **beliefs** about how other firms in the market respond strategically.
- This approach is known as the **Conjectural Variation Approach** or **Conjectural Variation Model**.

2. Conjectural variation model:

Estimating CV Parameters without data on MCs: Homogeneous Products

Conjectural Variation Model: Homogeneous Product Markets

- Consider an industry where, in period t, the inverse demand curve is:

$$p_t = P(Q_t, X_t^D)$$

and firms, indexed by i, have cost functions $C_i(q_{it})$.

- Each firm i chooses its output q_{it} to maximize profits:

$$\Pi_{it} = p_t q_{it} - C_i(q_{it})$$

- The first-order condition for profit maximization implies:

$$MR_{it} = MC_i(q_{it})$$

where the marginal revenue is:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\left(1 + \frac{dQ_{(-i)t}}{dq_{it}}\right) q_{it}$$

- The term $\dfrac{dQ_{(-i)t}}{dq_{it}}$ represents the **belief** that firm i has about how other firms will adjust their output if it changes its own output.
- We denote this belief as the **conjectural variation** of firm i, written as CV_i.

Conjectural Variations and Beliefs

- As researchers, we can consider different assumptions about firms' **beliefs** or **conjectural variations** (CV_{it}).
- An assumption on CV_{it} implies a specific **model of competition**.
- Different assumptions lead to different equilibrium outcomes:

$$q_{it}, \; Q_t, \text{ and } p_t$$

- However, not all conjectural variation assumptions are consistent with equilibrium.
- In fact, most assumptions about CV_{it} imply equilibria in which firms are **not rational** — that is, they hold beliefs about rivals' reactions that do not actually occur in equilibrium.

Conjectural Variations: Nash–Cournot Equilibrium

- In our model of firm competition, the **Nash conjecture** implies that:

$$CV_{it} \equiv \frac{\partial Q_{(-i)t}}{\partial q_{it}} = 0$$

- This conjecture leads to the **Cournot equilibrium** (or Nash–Cournot equilibrium).
- For every firm i, the perceived marginal revenue is:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\, q_{it}$$

- The first-order condition for profit maximization is therefore:

$$p_t + P'_Q(Q_t, X_t^D)\, q_{it} = MC_i(q_{it})$$

- This condition characterizes the **Cournot equilibrium**, where each firm treats the output of others as fixed when choosing q_{it}.

Conjectural Variations: Perfect Competition

- Are there other assumptions on firms' conjectural variations (CV_{it}) that are consistent with a **rational equilibrium**?
- Yes — certain conjectures generate **perfect competition** or **collusive (monopoly)** equilibria that are consistent with the outcomes they imply.
- **Perfect competition:** For every firm i, the conjecture is:

$$CV_{it} = -1$$

- This implies:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)[1 - 1]q_{it} = p_t$$

and thus the first-order condition:

$$p_t = MC_i(q_{it})$$

which characterizes the **perfect competition equilibrium**.

Conjectural Variations: Collusion

- Some beliefs can generate the **collusive (monopoly)** outcome as a rational equilibrium.
- **Collusion (Monopoly):** For every firm i, the conjecture is:

$$CV_{it} = N_t - 1$$

- This implies:

$$MR_{it} = p_t + P'_Q(Q_t, X_t^D)\, N_t q_{it}$$

- Hence, the equilibrium condition for each firm is:

$$p_t + P'_Q(Q_t, X_t^D)\, N_t q_{it} = MC_i(q_{it})$$

- When firms have constant and homogeneous marginal costs, these conditions simplify to:

$$p_t + P'_Q(Q_t, X_t^D)\, Q_t = MC$$

which is the equilibrium condition for the **monopoly (collusive or cartel)** outcome.

Conjectural Variations: Nature of Competition

- The value of firms' conjectural variations (CV_{it}) reflects the **nature of competition** in the market — whether it is Cournot, Perfect Competition, or Cartel (Monopoly).
- Summary of key cases:

$$\begin{aligned}
\text{Perfect Competition:} \quad & CV_{it} = -1, \quad MR_{it} = p_t \\
\text{Nash–Cournot:} \quad & CV_{it} = 0, \quad MR_{it} = p_t + P'_Q(Q_t)q_{it} \\
\text{Cartel (Monopoly):} \quad & CV_{it} = N_t - 1, \quad MR_{it} = p_t + P'_Q(Q_t)Q_t
\end{aligned}$$

- Hence, the conjectural variation parameter (CV) is closely related to the **degree of competition** and the resulting equilibrium prices and quantities.
- Interpretation:
  - If $CV < 0$: competition is **stronger than Cournot**.
  - The closer CV is to -1, the **more competitive** the market.
  - If $CV > 0$: competition is **weaker than Cournot**.
  - The closer CV is to $N_t - 1$, the **less competitive** (more collusive) the market.

Conjectural Variations: Nature of Competition [2]

- Interpreting the beliefs CV_{it} as an **index of competition** is correct and useful.
- However, it is important to note that for values of CV different from 1, 0, or $N_t - 1$, the resulting **conjectural variation equilibrium** is **not a rational equilibrium**.
- In such cases, firms' beliefs about how rivals respond are not consistent with actual equilibrium behavior.
- We can think of CV_{it} as firms' **beliefs formed dynamically over time** — evolving through firms' interactions, observation, and learning.
- In this sense, conjectural variations can be interpreted as emerging from a **dynamic game of learning and adjustment**.

Conjectural Variation: Estimation

- Consider a homogeneous product industry where the researcher observes firms' quantities, marginal costs, and market prices over T periods:

  $$\text{Data} = \{p_t,\ MC_{it},\ q_{it}\} \quad \text{for } i = 1, 2, \ldots, N_t \text{ and } t = 1, 2, \ldots, T$$

- Assuming that each firm i chooses output q_{it} to maximize profits given its belief CV_{it}, the following condition holds:

  $$p_t + P'_Q(Q_t, X_t^D)[1 + CV_{it}]q_{it} = MC_{it}$$

- Solving for the conjectural variation gives:

  $$CV_{it} = \frac{p_t - MC_{it}}{P'_Q(Q_t, X_t^D)q_{it}} - 1$$

- Alternatively, using the definition of the demand elasticity η_t:

  $$CV_{it} = \left(\frac{p_t - MC_{it}}{p_t} \cdot \frac{Q_t}{q_{it}}\right)|\eta_t| - 1$$

  where η_t is the (absolute value of the) market demand elasticity.

Conjectural Variation: Estimation [2]

- Recall the expression for the conjectural variation:

$$CV_{it} = \left(\frac{(p_t - MC_{it})/p_t}{q_{it}/Q_t} \right) |\eta_t| - 1$$

- This equation shows that, given data on quantities, prices, demand, and marginal costs, we can identify the firms' **beliefs (CV_{it})** that are consistent with profit maximization.
- Let

$$\left(\frac{(p_t - MC_{it})/p_t}{q_{it}/Q_t} \right)$$

  be denoted as the **Lerner-index-to-market-share ratio** for firm i.
- Interpretation:
  - If Lerner-index-to-market-share ratios are close to zero, then CV_{it} will be close to 1, unless the absolute demand elasticity $|\eta_t|$ is very large.
  - If the Lerner-index-to-market-share ratios are large (i.e., larger than the inverse demand elasticity), then estimated CV_{it} values will be greater than zero — suggesting behavior less competitive than Cournot.

Conjectural Variation: Estimation [3]

- Recall:

$$CV_{it} = \left(\frac{(p_t - MC_{it})/p_t}{q_{it}/Q_t} \right) |\eta_t| - 1$$

- Part of the observed variation in CV_{it} may arise from **estimation error** in the demand and marginal cost functions.
- To formally test the value of CV_{it}, we must account for this estimation uncertainty.
- Let $\overline{CV}$ denote the **sample mean** of all estimated CV_{it} values.
- Under the **null hypothesis of Cournot competition**, we have:

$$H_0 : CV_{it} = 0 \quad \forall (i, t)$$

- Assuming asymptotic normality:

$$\overline{CV} \sim N(0, \sigma^2)$$

- We can estimate $\hat{\sigma}$ and construct a **t-test statistic**:

$$t = \frac{\overline{CV}}{\hat{\sigma}}$$

  to test whether the conjectural variations are significantly different from zero.

3. Conjectural variation model:

Estimating CV Parameters without data on MCs: Homogeneous Products

Estimating CV Parameters without Data on MCs

- So far, we have considered the estimation of **conjectural variation (CV)** parameters when the researcher knows both the demand system and firms' marginal costs.
- We now consider the case where the researcher knows the **demand function**, but does not observe firms' marginal costs.
- In this setting, **identification of CVs also requires identification of marginal costs (MCs)**.
- Under certain conditions, it is possible to **jointly identify** the CV parameters and the MCs using:
  - The **marginal conditions of optimality**, and
  - The **estimated demand system**.

Data

- The researcher observes the following data:

$$\text{Data} = \left\{ P_t,\ q_{it},\ X_t^D,\ X_t^{MC} : i = 1, \ldots, N_t;\ t = 1, \ldots, T \right\}$$

- Where:
  - P_t — market price at period t,
  - q_{it} — quantity produced by firm i at period t,
  - X_t^D — variables affecting **consumer demand** (e.g., average income, population),
  - X_t^{MC} — variables affecting **marginal costs** (e.g., input prices, technology indicators).

Model: Demand and Marginal Costs

- Consider the linear (inverse) demand equation:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t + \varepsilon_t^D,$$

  where $\alpha_2 > 0$ and ε_t^D is an unobservable demand shock.
- The marginal cost function for firm i is given by:

$$MC_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2 q_{it} + \varepsilon_{it}^{MC},$$

  where $\beta_2 \geq 0$ and ε_{it}^{MC} is an unobservable cost shock.
- Parameters:
  - $\alpha_0, \alpha_1, \alpha_2$ — demand parameters,
  - $\beta_0, \beta_1, \beta_2$ — cost structure parameters.

Model: Profit Maximization

- Profit maximization implies:

$$MR_{it} = MC_{it} \quad \Rightarrow \quad P_t + \frac{dP_t}{dQ_t}[1 + CV_{it}]q_{it} = MC_{it}$$

- From the inverse demand function, $\frac{dP_t}{dQ_t} = -\alpha_2$. Substituting into the condition gives:

$$P_t - \alpha_2[1 + CV_{it}]q_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2 q_{it} + \varepsilon_{it}^{MC}$$

- Rearranging, we obtain:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + [\beta_2 + \alpha_2(1 + CV_{it})]q_{it} + \varepsilon_{it}^{MC}$$

- This equation represents the **marginal condition for profit maximization**.
- We now assume that $CV_{it} = CV$ for all (i, t) in the data — i.e., a common conjectural variation across firms and time.

Complete Structural Model

- The structural equations of the model are:
  Demand equation:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t + \varepsilon_t^D$$

  First-order condition (F.O.C.):

$$P_t = \beta_0 + \beta_1 X_t^{MC} + [\beta_2 + \alpha_2(1 + CV)]q_{it} + \varepsilon_{it}^{MC}$$

- Key question: Can we identify (i.e., estimate consistently, without asymptotic bias) the **conjectural variation parameter** CV using this model and available data?
- The answer is **No**. In this specification, we cannot separately identify CV and the marginal cost slope parameter β_2 because they enter the F.O.C. additively through the term:

$$[\beta_2 + \alpha_2(1 + CV)]$$

- However, we will see that a **simple modification** of the model allows for the **separate identification** of CV and the marginal cost parameters.

Identification of Demand Parameters

- The demand equation is:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t + \varepsilon_t^D$$

- **Endogeneity problem:** In equilibrium,

$$\text{Cov}(Q_t, \varepsilon_t^D) \neq 0$$

  because the equilibrium quantity Q_t depends on unobserved demand shocks ε_t^D.
- The model, however, provides a **valid instrument** to estimate demand.
- In equilibrium, Q_t depends on X_t^{MC}, but X_t^{MC} does not appear in the demand equation.
- If X_t^{MC} is uncorrelated with ε_t^D, then X_t^{MC} satisfies all the conditions for being a **valid instrument** for Q_t.
- Therefore, parameters α_0, α_1, and α_2 can be identified and consistently estimated using an **IV estimator**.

Identification of CV and MCs

- The first-order condition (F.O.C.) is:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + [\beta_2 + \alpha_2(1 + CV)]q_{it} + \varepsilon_{it}^{MC}$$

- **Endogeneity problem:** In equilibrium,

$$\text{Cov}(q_{it}, \varepsilon_{it}^{MC}) \neq 0$$

  because the firm's output q_{it} depends on unobserved cost shocks ε_{it}^{MC}.
- The model provides a **valid instrument** for estimation.
- In equilibrium, q_{it} depends on X_t^D, but X_t^D does not appear in the F.O.C.
- If X_t^D is uncorrelated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a **valid instrument** for q_{it}.
- Therefore, parameters β_0, β_1, and

$$\gamma \equiv \beta_2 + \alpha_2(1 + CV)$$

  can be identified and consistently estimated using an **IV estimator**.

The Identification Problem

- The first-order condition (F.O.C.) can be written as:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma q_{it} + \varepsilon_{it}^{MC}$$

- We can identify the composite parameter γ, where:

$$\gamma \equiv \beta_2 + \alpha_2(1 + CV)$$

  and the slope of the inverse demand function, α_2.

- However, knowledge of γ and α_2 is **not sufficient** to separately identify CV and the slope of the marginal cost function, β_2.
- For example, suppose $\gamma = 1$ and $\alpha_2 = 0.4$, so that:

$$1 = \beta_2 + 0.4(1 + CV)$$

- This equation can be satisfied by several (β_2, CV) pairs:

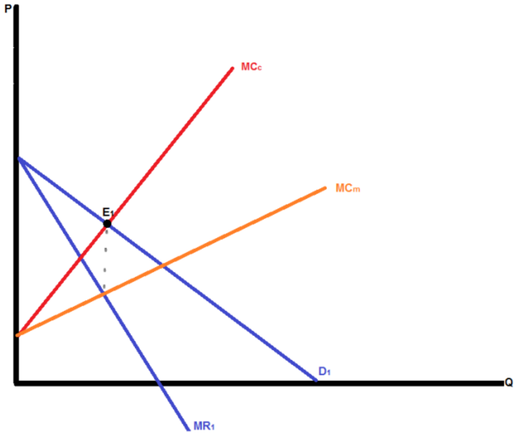
$$\begin{aligned} \text{[Perfect Competition]:} \quad & CV = 1, \ \beta_2 = 1.0 \\ \text{[Cournot]:} \quad & CV = 0, \ \beta_2 = 0.6 \\ \text{[Cartel, with } N = 3\text{]:} \quad & CV = N - 1 = 2, \ \beta_2 = 0.2 \end{aligned}$$

- $\Rightarrow$ Therefore, **without additional variation or restrictions**, CV and β_2 are **not separately identified**.

The Identification Problem [2]

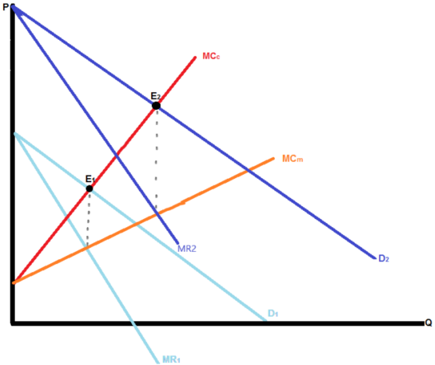
- The IV estimator identifies the marginal cost (MC) parameters by using the instrument X_t^D, which shifts the demand curve.
- When we make an **assumption about the form of competition**, these demand shifts allow us to trace out the marginal cost curve — i.e., to identify the MC parameters.
- However, **without specifying the form of competition**, shifts in the demand alone are **not sufficient** to separately identify the MC and the conjectural variation (CV).
- Let $\hat{q}_{it}(X_t^D)$ denote the part of q_{it} explained by the demand shifters X_t^D.
- When X_t^D varies, we observe a positive correlation between P_t and $\hat{q}_{it}(X_t^D)$.
- But the **magnitude** of this correlation can be rationalized by different parameter combinations:
  - Either a **zero or negative** *CV* and a **large, positive** β_2, or
  - A **positive** *CV* and a **small or zero** β_2.

The identification problem [3]



- Note that point E_1 could represent an equilibrium either for:
  - A **perfectly competitive industry** with marginal cost MC^c, or
  - A **monopolist** (or collusive industry) with marginal cost MC^m.

The identification problem [4]

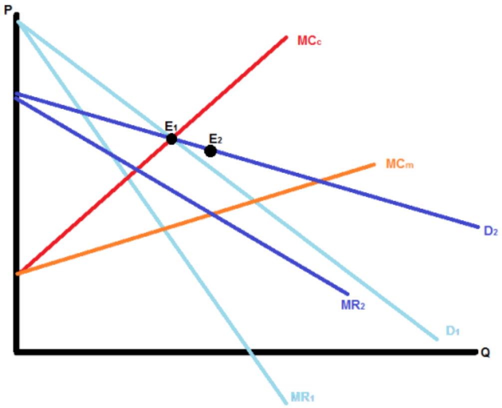


- The current demand modle and the IV parallely shifts the Demand curve (and hence the MR curve).
- In the new equilibrium E2, it can still be supported by
  - A **perfectly competitive industry** with marginal cost MC^c, or
  - A **monopolist** (or collusive industry) with marginal cost MC^m.

Solving the Identification Problem

- Solving the **identification problem** requires generalizing the demand system so that changes in exogenous variables do more than simply produce **parallel shifts** of the demand and marginal revenue (MR) curves.
- In particular, we need to include additional exogenous variables that can **rotate** the demand curve — changing its slope as well as its level.
- These variables are called **Demand Rotators**.
- **Demand Rotators** are exogenous factors that affect the **slope** of the demand curve, rather than just its position.

Solving the Identification Problem [2]



- Now, rotate the demand curve to D_2, with corresponding marginal revenue curve MR_2.
- The competitive equilibrium remains at E_1, but the monopoly equilibrium moves to E_2.

Solving the Identification Problem [3]

- Consider the following generalized demand equation:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t - \alpha_3 (R_t Q_t) + \varepsilon_t^D$$

- R_t is an **observable variable** that affects the **slope of the demand curve**.
- Economically, R_t may represent:
  - The price of a **substitute** or **complement** good,
  - A variable influencing **consumer price sensitivity** or **market substitutability**.
- The key condition for identification is:

$$\alpha_3 \neq 0$$

- This means that when R_t varies, it causes a **rotation**—not just a parallel shift—of the demand curve.
- Such variation in the slope of demand allows for the **separate identification** of the conjectural variation (CV) and marginal cost parameters.

Solving the Identification Problem [4]

- From the generalized demand model:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t - \alpha_3 (R_t Q_t) + \varepsilon_t^D$$

- The slope of the inverse demand curve is:

$$\frac{dP_t}{dQ_t} = -(\alpha_2 + \alpha_3 R_t)$$

- The first-order condition (F.O.C.) for profit maximization is:

$$P_t + \frac{dP_t}{dQ_t}[1 + CV]q_{it} = MC_{it}$$

- Substituting for $\frac{dP_t}{dQ_t}$, we obtain:

$$P_t - (\alpha_2 + \alpha_3 R_t)[1 + CV]q_{it} = MC_{it}$$

- Or equivalently:

$$P_t = MC_{it} + (\alpha_2 + \alpha_3 R_t)[1 + CV]q_{it}$$

- This specification allows identification of CV because variation in R_t (the **demand rotator**) changes the slope of the F.O.C. relationship.

Solving the Identification Problem [5]

Combining the first-order condition (F.O.C.) with the marginal cost (MC) function

$$MC_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2 q_{it} + \varepsilon_{it}^{MC},$$

we have:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \beta_2 q_{it} + (\alpha_2 + \alpha_3 R_t)[1 + CV]\, q_{it} + \varepsilon_{it}^{MC}.$$

This can be represented by the following regression model:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 q_{it} + \gamma_2 (R_t q_{it}) + \varepsilon_{it}^{MC},$$

with

$$\gamma_1 = \beta_2 + \alpha_2[1 + CV] \quad \text{and} \quad \gamma_2 = \alpha_3[1 + CV].$$

Solving the Identification Problem [6]

- The structural equations of the model are:
  Demand:

  $$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t - \alpha_3 (R_t Q_t) + \varepsilon_t^D$$

  First-order condition (F.O.C.):

  $$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 q_{it} + \gamma_2 (R_t q_{it}) + \varepsilon_{it}^{MC}$$

- In this formulation:

  $$\gamma_1 = \beta_2 + \alpha_2(1 + CV)$$

  $$\gamma_2 = \alpha_3(1 + CV)$$

- Since both γ_1 and γ_2 can be identified from the data, and α_2, α_3 are identified from the demand equation, we can now **separately identify**:
  - The conjectural variation parameter, CV, and
  - The slope of the marginal cost function, β_2.
- $\Rightarrow$ Variation in R_t (the **demand rotator**) provides the additional information needed for full identification.

Identification of Demand Parameters

- The demand equation is:

$$P_t = \alpha_0 + \alpha_1 X_t^D - \alpha_2 Q_t - \alpha_3 (R_t Q_t) + \varepsilon_t^D$$

- **Endogeneity problem:**

$$\text{Cov}(Q_t, \varepsilon_t^D) \neq 0$$

  since the equilibrium quantity Q_t depends on the unobserved demand shock ε_t^D.
- The model provides a **valid instrument** for identification.
- In equilibrium, Q_t depends on X_t^{MC}, but X_t^{MC} does not appear in the demand equation.
- If X_t^{MC} is uncorrelated with ε_t^D, then X_t^{MC} satisfies all the conditions for being a **valid instrument** for Q_t.
- Therefore, the parameters α_0, α_1, α_2, and α_3 are **identified and consistently estimated** using an **IV estimator**.

Identification of CV and MCs

- The first-order condition (F.O.C.) is:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 q_{it} + \gamma_2 (R_t q_{it}) + \varepsilon_{it}^{MC}$$

- **Endogeneity problem:**

$$\text{Cov}(q_{it}, \varepsilon_{it}^{MC}) \neq 0$$

  since firm output q_{it} depends on unobserved cost shocks ε_{it}^{MC}.
- The model provides a **valid instrument** for estimation.
- In equilibrium, q_{it} depends on X_t^D, but X_t^D does not appear in the F.O.C.
- If X_t^D is uncorrelated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a **valid instrument** for q_{it}.
- Therefore, the parameters

$$\beta_0,\ \beta_1,\ \gamma_1,\ \text{and}\ \gamma_2$$

  are **identified and can be consistently estimated** using an **IV estimator**.

Identification of CV and MCs [2]

- Recall the first-order condition (F.O.C.):

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 q_{it} + \gamma_2 (R_t q_{it}) + \varepsilon_{it}^{MC}$$

- From the model structure, we know that:

$$\gamma_1 = \beta_2 + \alpha_2[1 + CV]$$

$$\gamma_2 = \alpha_3[1 + CV]$$

- **Step 1: Identification of** CV

$$CV = \frac{\gamma_2}{\alpha_3} - 1$$

- **Step 2: Identification of the slope of MC (**β_2**)**

$$\beta_2 = \gamma_1 - \alpha_2[1 + CV]$$

- $\Rightarrow$ Given the parameters (γ_1, γ_2) from the F.O.C. and (α_2, α_3) from the demand equation, both the **conjectural variation parameter (**CV**)** and the **marginal cost slope (**β_2**)** are **separately identified**.

Identification of CV and MCs [3]

- Recall the first-order condition (F.O.C.):

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 q_{it} + \gamma_2 (R_t q_{it}) + \varepsilon_{it}^{MC}$$

- From the model:

$$\gamma_2 = \alpha_3 [1 + CV]$$

- **Identification of CV:**

$$1 + CV = \frac{\gamma_2}{\alpha_3}$$

  This ratio measures the relative sensitivity of P_t:
  - In the F.O.C. with respect to $(R_t q_{it})$, and
  - In the demand equation with respect to $(R_t Q_t)$.
- **Example:** Let $\alpha_3 = 0.5$ and $N = 3$.

$$\begin{aligned}
\text{Perfect Competition:} \quad & CV = 1 \;\Rightarrow\; \frac{\gamma_2}{\alpha_3} = 0 \\
\text{Cournot:} \quad & CV = 0 \;\Rightarrow\; \frac{\gamma_2}{\alpha_3} = \frac{1}{0.5} = 2 \\
\text{Cartel } (N = 3): \quad & CV = N - 1 = 2 \;\Rightarrow\; \frac{\gamma_2}{\alpha_3} = \frac{2}{0.5} = 4
\end{aligned}$$

- $\Rightarrow$ The larger γ_2 / α_3, the **weaker the competition**.

4. Conjectural variation model:

Estimating CV Parameters under Differentiated Products

CVs with Product Differentiation

- We illustrate the conjectural variation (CV) approach in a simplified model with **two firms**, each producing a **differentiated product**.
- This framework can be easily extended to N multiproduct firms (see Nevo, *Economics Letters*, 1998).
- **Industry setup:**
  - Firms: $i = 1, 2$
  - Each firm produces and sells one product.
- **Profit of firm i:**

$$\Pi_i = p_i q_i - C_i(q_i)$$

- **Demand:** The market has a Logit structure with total market size H:

$$q_1 = Hs_1 = \frac{\exp\{\beta x_1 - \alpha p_1\}}{1 + \exp\{\beta x_1 - \alpha p_1\} + \exp\{\beta x_2 - \alpha p_2\}}$$

- Similarly:

$$q_2 = Hs_2 = \frac{\exp\{\beta x_2 - \alpha p_2\}}{1 + \exp\{\beta x_1 - \alpha p_1\} + \exp\{\beta x_2 - \alpha p_2\}}$$

Conjectural Variation

- Recall the market share function for product 1:

$$s_1 = \frac{\exp\{\beta x_1 - \alpha p_1\}}{1 + \exp\{\beta x_1 - \alpha p_1\} + \exp\{\beta x_2 - \alpha p_2\}}$$

- The market share s_1 depends on p_1 through two different channels:

$$\begin{cases} \text{(i) Direct effect: } p_1 \to s_1 \\ \text{(ii) Indirect effect: } p_1 \to p_2 \to s_1 \end{cases}$$

- To capture this, we must distinguish between:
  - The **total derivative**:

$$\frac{ds_1}{dp_1} = \frac{\partial s_1}{\partial p_1} + \frac{\partial s_1}{\partial p_2}\frac{dp_2}{dp_1}$$

  - and the **partial derivatives**:

$$\frac{\partial s_1}{\partial p_1} \quad \text{and} \quad \frac{\partial s_1}{\partial p_2}$$

- The partial derivatives hold the other price constant, while the total derivative accounts for the strategic reaction of firm 2,

Conjectural Variation [2]

- Starting from the total differential of s_1:

$$ds_1 = \frac{\partial s_1}{\partial p_1} dp_1 + \frac{\partial s_1}{\partial p_2} dp_2$$

- Dividing by dp_1, we obtain:

$$\frac{ds_1}{dp_1} = \frac{\partial s_1}{\partial p_1} + \frac{\partial s_1}{\partial p_2} \frac{dp_2}{dp_1}$$

- The term

$$\frac{dp_2}{dp_1}$$

  represents the **belief** or **conjecture** of firm 1 regarding how firm 2 adjusts its price in response to a change in p_1.
- We denote this as:

$$CV_1 = \frac{dp_2}{dp_1}$$

- CV_1 is therefore **Firm 1's Conjectural Variation** — it captures firm 1's perceived degree of strategic interdependence in pricing.

Conjectural Variation [3]

- For the standard Logit demand model:

$$\frac{\partial s_1}{\partial p_1} = -\alpha s_1(1 - s_1), \quad \frac{\partial s_1}{\partial p_2} = \alpha s_1 s_2$$

- Therefore, the total derivative of s_1 with respect to p_1 is:

$$\frac{ds_1}{dp_1} = \frac{\partial s_1}{\partial p_1} + \frac{\partial s_1}{\partial p_2}\frac{dp_2}{dp_1}$$

- Substituting and using $CV_1 = \frac{dp_2}{dp_1}$:

$$\frac{ds_1}{dp_1} = -\alpha s_1(1 - s_1) + \alpha s_1 s_2 CV_1 = -\alpha s_1(1 - s_1 - s_2 CV_1)$$

- Plugging this into Firm 1's first-order condition:

$$p_1 - MC_1 = \frac{s_1}{\alpha s_1(1 - s_1 - s_2 CV_1)}$$

- Simplifying:

$$p_1 - MC_1 = \frac{1}{\alpha(1 - s_1 - s_2 CV_1)}$$

Different Conjectures – Forms of Competition

- Recall the general expression:

$$p_1 - MC_1 = \frac{1}{\alpha(1 - s_1 - s_2 CV_1)}$$

- **Nash–Bertrand Competition:** When each firm takes the rival's price as fixed ($CV_1 = 0$),

$$p_1 - MC_1 = \frac{1}{\alpha(1 - s_1)}$$

  Firm 1 behaves non-cooperatively — only its own share elasticity matters.

- **Collusion between Firms 1 and 2:** When both firms act jointly to maximize their combined profits ($CV_1 = 1$),

$$p_1 - MC_1 = \frac{1}{\alpha(1 - s_1 - s_2)}$$

  This corresponds to the first-order condition for a single firm producing both products 1 and 2.

- $\Rightarrow$ The conjectural variation parameter CV_1 provides a continuous measure of the **degree of market coordination** — ranging from independent pricing ($CV_1 = 0$) to full collusion ($CV_1 = 1$).

Extension to N Firms

- When there are $N > 2$ firms, the conjectural variation parameter

$$CV_{i\to j} = \frac{dp_j}{dp_i}$$

represents **firm i's conjecture** about how firm j adjusts its price in response to a change in p_i.

- For the Logit demand model, the first-order condition (F.O.C.) for profit maximization of firm i becomes:

$$p_i - MC_i = \frac{1}{\alpha\left(1 - s_i - \sum_{j\neq i} s_j\, CV_{i\to j}\right)}$$

- Interpretation:
  - The markup $(p_i - MC_i)$ depends on firm i's own market share s_i, the shares of competitors s_j, and its beliefs $CV_{i\to j}$ about their price responses.
  - Higher (positive) conjectures $CV_{i\to j}$ imply **less competitive behavior** — firms internalize rivals' reactions.
  - Negative conjectures correspond to **aggressive competition** (strategic substitutability).
- $\Rightarrow$ The conjectural variation framework nests **Bertrand, Cournot, and collusive** behavior as special cases of the same general conduct model.

Identification of Collusion

- Recall the general first-order condition for firm i:

$$p_i - MC_i = \frac{1}{\alpha\left(1 - s_i - \sum_{j \neq i} s_j \, CV_{i \to j}\right)}$$

- As in the homogeneous product case, we must distinguish two situations:
  - **Case 1:** Marginal costs (MC_i) are known.
  - **Case 2:** Marginal costs (MC_i) are unknown.
- Empirical research often focuses on the **identification of collusion**, based on assumptions about firms' conjectures:

$$\begin{cases} CV_{i \to j} \in \{0, 1\} & \text{(Non-cooperative vs. collusive conduct)} \\ CV_{i \to j} = CV_{j \to i} & \text{(Symmetric interaction)} \end{cases}$$

- Under mild regularity conditions (e.g., demand instruments, symmetric conjectures, and identifiable market shares), these conjectural variations can be **empirically identified**.